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2nd MICROLOCALISATION AND CONICAL REFRACTION

by Nobuyuki TOSE

Contents

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1. Introduction.

The phenomenon of conical refraction has long been observed by physicians : a ray splits into a cone by a biaxial crystal. This fact is attributed by mathematicians to the non-uniformity of the multiplicity of Maxwell equation in the crystal. Microlocal analysis of the conical refraction is studied in C^∞ case by Melrose-Uhlmann [8] and in the real analytic case by Laubin [5], [6].

In this paper, we employ the theory of 2-microlocalisation developed by Kashiwara and Laurent (see [2], [4]) and gain a new insight about the conical refraction.

Explicitly, P is a microdifferential operator defined in a neighborhood of $\rho_0 \in \sqrt{-1}\mathring{T}^*\mathbf{R}^n$, which satisfies the following conditions :

(1.1) P has the real principal symbol p.

Let $\Sigma_1 = \{\rho \in \sqrt{-1}\,T^*\mathbf{R}^n;\ p(\rho)=0\}$ and $\Sigma_2 = \{\rho \in \Sigma_1;\ dp(\rho)=0\}$.

(1.2) Σ_2 is a regular involutory submanifold of codimension $d \geqslant 3$ through ρ_0.

(1.3) Hess $(p)(\rho)$ has rank d and positivity 1 if $\rho \in \Sigma_2$.

(1.4) P has regular singularities along $\Sigma_2^{\mathbf{C}}$ in the sense of Kashiwara-Oshima [3], where $\Sigma_2^{\mathbf{C}}$ denotes a complexification of Σ_2 in $T^*\mathbf{C}^n$.

Our main interest is the propagation of singularities on Σ_2 for the equation

$$(1.5) \qquad Pu = 0.$$

First we transform the equation (1.5) into

$$(1.6) \qquad P_0 u = (D_1^2 - \sum_{i,j=2}^{d} A^{ij}(x,D)D_iD_j + (\text{lower}))u = 0$$

by a real quantized contact transformation, where A^{ij} are of order 0 with $(\sigma(A^{ij}))$ positive definite. This fact is already shown by Melrose-

Uhlmann [8] and Laubin [5], [6]. We remark that in this case

$$\Sigma_2 = \Sigma = \{(x,\sqrt{-1}\xi\, dx); \xi_1 = \xi_2 = \cdots = \xi_d = 0\}.$$

Secondly we study the equation (1.6) 2-microlocally along Σ. After transforming (1.6) by a suitable quantized homogeneous bicanonical transformation, we give its 2-microlocal canonical form as

$$\text{(1.7)} \qquad D_1 u = 0$$

defined in a neighborhood of $(0;\sqrt{-1}\,dx_n;\sqrt{-1}\,dx_d) \in T^*_{\Sigma}\tilde{\Sigma}$. Then we can easily obtain a theorem about the propagation of 2-microlocal singularities and generalize the result of Laubin [5], [6] that treats the propagation of microlocal singularities.

The author would like to express his gratitude to Prof. H. Komatsu for guidance and encouragement. He wishes to dedicate this paper to his grand professor K. Yosida for his 77th birthday.

2. Preliminary.

2.1. 2-microdifferential operators.

We review the theory of 2-microdifferential operators defined by Y. Laurent [4].

Let X be an open subset of $\mathbf{C}^{n+d}$ and T*X be its cotangent bundle. We take a coordinate of X as (w,z) with $w \in \mathbf{C}^n$ and $z \in \mathbf{C}^d$. Then $p = (w,z;\theta\, dw + \zeta\, dz)$ denotes a point of T*X with $\theta \in \mathbf{C}^n$ and $\zeta \in \mathbf{C}^d$. T*X is endowed with the sheaf $\mathscr{E}_X$ of microdifferential operators defined by Sato-Kawai-Kashiwara [9]. See also Schapira [10] for details about $\mathscr{E}_X$.

Hereafter in this section 2.1, Λ is a regular involutory submanifold of $\dot{T}^*X$ $(= T^*X \backslash X)$

$$\text{(2.1)} \qquad \Lambda = \{(w,z;\theta\, dw + \zeta\, dz); \zeta = 0\}.$$

We identify Λ with a submanifold of $\Lambda \times \Lambda$ through

$$T^*X \simeq T^*_X X \times X \hookrightarrow T^*X \times X.$$

By definition, $\tilde{\Lambda}$ is the union of bicharacteristics of $\Lambda \times \Lambda$ that pass through Λ. We take a coordinate of $T^*_\Lambda\tilde{\Lambda}$ as $(w,z;\theta\, dw; z^*\, dz)$ with $(w,z;\theta\, dw) \in \Lambda$ and $z^* \in \mathbf{C}^d$.

The sheaf $\mathscr{E}^{2,\infty}_\Lambda$ of 2-microdifferential operators of infinite order is constructed $T^*_\Lambda\tilde{\Lambda}$ by Y. Laurent [4].

DEFINITION 2.1. – *For an open subset* U *of* $T^*_\Lambda\tilde{\Lambda}$, *a formal sum* $\sum_{(i,j)\in \mathbf{Z}^2} P_{ij}(w,z,D_w,D_z)$ *belongs to* $\mathscr{E}^{2,\infty}_\Lambda(U)$ *if and only if the following conditions are satisfied :*

(2.2) $P_{ij}(w,z,\theta,z^*)$ *is holomorphic on* U *and is homogeneous of order* j *with respect* (θ,z^*) *and order* i *with respect to* z^*.

(2.3) *For any compact subset* K *of* U, *there exists a positive number* C_K. *For any positive* ε *and a compact subset* K, *we can take a positive number* $C_{\varepsilon,K}$ *such that*

$$\sup_K |P_{i,i+k}| \leqslant \begin{cases} C_{\varepsilon,K}\varepsilon^{i+k}/i!\,k! & (i,k\geqslant 0) \\ C_{\varepsilon,K}^{-k}\varepsilon^{i}(-k)!/i! & (i\geqslant 0, k<0) \\ C_{\varepsilon,K}\varepsilon^{k}C_K^{-i}(-i)!/k! & (i<0,\ k\geqslant 0) \\ C_K^{-i-k}(-i)!(-k)! & (i,k<0). \end{cases}$$

Y. Laurent also defined the sheaf $\mathscr{E}^{2,(r,1)}_\Lambda$ of 2-microdifferential operators of finite order of type $(r,1)$.

DEFINITION 2.2. – *Let* U *be an open subset of* $T^*_\Lambda\tilde{\Lambda}$ *and* P *be an element of* $\mathscr{E}^{2,(r,1)}_\Lambda(U)$. *For* $r(>1) \in \mathbf{Q}\cup\{\infty\}$ *and* $(i_0,j_0) \in \mathbf{Z}^2$,

$$P = \sum_{(i,j)} P_{ij}(w,z,D_z,D_w) \in \mathscr{E}^{2,(r,1)}_\Lambda[i_0,j_0]$$

if and only if

(2.4) $$P_{ij} \equiv 0 \text{ if } \frac{1}{r}\, i + (j-i) > \frac{1}{r}\, i_0 + (j_0 - i_0) \text{ or } j > j_0.$$

We put

(2.5) $$\mathscr{E}^{2,(r,1)}_\Lambda = \bigcup_{i,j} \mathscr{E}^{2,(r,1)}_\Lambda[i,j].$$

For an element of $\mathscr{E}_\Lambda^{2,(r,1)}$, *the principal symbol of type* $(r,1)$ *is defined by*

$$\sigma_\Lambda^{(r,1)}(P) = P_{i_0,j_0}, \tag{2.6}$$

where P *is not a section of* $\mathscr{E}_\Lambda^{2,(r,1)}[i,j]$ *that is strictly smaller than* $\mathscr{E}_\Lambda^{2,(r,1)}[i_0,j_0]$.

See Y. Laurent [4], for details about 2-microdifferential operators.

2.2. Bisymplectic structure of $T_\Lambda^*\tilde{\Lambda}$.

Y. Laurent introduced in [4] the transformation theory for 2-microdifferential operators, which is wider than the quantized contact transformation. We review the notion of quantized bicanonical transformation in this § 2.2.

Let X be a complex manifold and Λ be a regular involutory submanifold in $\mathring{T}^*X$. The regular involutory submanifold in (2.1) is denoted by Λ_0 in § 2.2. We identify Λ with a submanifold of $\Lambda \times \Lambda$ in the same way as § 2.1. Then $\tilde{\Lambda}$ is the union of all bicharacteristic leaves of $\Lambda \times \Lambda$ issued from Λ.

$T_\Lambda^*\tilde{\Lambda}$ has a canonical 1-form $\omega_\Lambda = p^{-1}\omega_X$. Here $p : T_\Lambda^*\tilde{\Lambda} \to \Lambda \to T^*X$ and ω_X is the canonical 1-form of T^*X. In case $\Lambda = \Lambda_0$, ω_Λ is expressed by coordinates as

$$\omega_\Lambda = \sum_{j=1}^{n} \theta_j \, dw_j. \tag{2.7}$$

We also define the canonical 2-form $\Omega_\Lambda = d\omega_\Lambda$.

Ω_Λ endows a scalar product on $T(T_\Lambda^*\tilde{\Lambda})$. We put its kernel as $T_{\mathrm{rel}}(T_\Lambda^*\tilde{\Lambda})$. We define an exact sequence

$$0 \to T_{\mathrm{rel}}T_\Lambda^*\tilde{\Lambda} \to TT_\Lambda^*\tilde{\Lambda} \to \tilde{T}T_\Lambda^*\tilde{\Lambda} \to 0 \tag{2.8}$$

and its dual

$$0 \leftarrow T_{\mathrm{rel}}^*T_\Lambda^*\tilde{\Lambda} \leftarrow T^*T_\Lambda^*\tilde{\Lambda} \leftarrow \tilde{T}^*T_\Lambda^*\tilde{\Lambda} \leftarrow 0. \tag{2.9}$$

We can take a section ω_Λ^r of $T_{\mathrm{rel}}^*T_\Lambda^*\tilde{\Lambda}$ canonically, which is called the relative canonical 1-form and constructed in the following way. We also define the relative 2-form $\Omega_\Lambda^r = d\omega_\Lambda^r$. We can show

$$\tilde{T}^*(T_\Lambda^*\tilde{\Lambda}) \simeq (T_\Lambda^*\Lambda \times \Lambda) \underset{\tilde{\Lambda}}{\times} T_\Lambda^*\tilde{\Lambda}. \tag{2.10}$$

See Lemma 2.9.8 of Y. Laurent [4]. On the other hand we derive the diagram

$$
(2.11)\qquad
\begin{array}{ccccccccc}
 & & & & 0 & & & & \\
 & & & & \downarrow & & & & \\
 & & & & (T^*_{\tilde\Lambda}\Lambda\times\Lambda)\underset{\tilde\Lambda}{\times}\Lambda & & & & \\
 & & & & & & 0 & & \\
 & & & & \downarrow & & \downarrow & & \\
0 & \to & T^*\Lambda & \to & (T^*\Lambda\times\Lambda)\underset{\tilde\Lambda}{\times}\Lambda & \to & T^*\Lambda & \to & 0 \\
 & & & & \downarrow & & \downarrow & & \\
0 & \to & T^*_\Lambda\tilde\Lambda & \to & (T^*_\Lambda\tilde\Lambda)\underset{\tilde\Lambda}{\times}\Lambda & \to & T^*\Lambda & \to & 0 \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & 0 & & 0 & &
\end{array}
$$

from $\tilde\Lambda \hookrightarrow \Lambda\times\Lambda$, $\tilde\Lambda \hookleftarrow \Lambda \hookrightarrow \Lambda\times\Lambda$.

By (2.11) we get the exact sequence on Λ:

$$
(2.12)\qquad 0 \to (T^*_{\tilde\Lambda}\Lambda\times\Lambda)\underset{\tilde\Lambda}{\times}\Lambda \to T^*\Lambda \to T^*_\Lambda\tilde\Lambda \to 0.
$$

Moreover from $T^*_\Lambda\tilde\Lambda \to \Lambda$, the injection

$$
(2.13)\qquad (T^*\Lambda)\underset{\Lambda}{\times} T^*_\Lambda\tilde\Lambda \to T^*T^*_\Lambda\tilde\Lambda
$$

follows. After all, we have the diagram

$$
(2.14)\qquad
\begin{array}{ccccccccc}
 & & & & & & 0 & & \\
 & & & & & & \downarrow & & \\
0 & \to & T^*_{\tilde\Lambda}\Lambda\times\Lambda\underset{\tilde\Lambda}{\times}T^*_\Lambda\tilde\Lambda & \to & T^*\Lambda\underset{\Lambda}{\times}T^*_\Lambda\tilde\Lambda & \to & T^*_\Lambda\tilde\Lambda\times T^*_\Lambda\tilde\Lambda & \to & 0 \\
 & & \wr\! & & \downarrow & & & & \\
0 & \to & T^*_{\tilde\Lambda}\Lambda\times\Lambda\times T^*_\Lambda\tilde\Lambda & \to & T^*T^*_\Lambda\tilde\Lambda & \to & T^*_{\mathrm{rel}}T^*_\Lambda\tilde\Lambda & \to & 0
\end{array}
$$

and derive the injective morphism

$$
(2.15)\qquad T^*_\Lambda\tilde\Lambda\underset{\Lambda}{\times}T^*_\Lambda\tilde\Lambda \to T^*_{\mathrm{rel}}T^*_\Lambda\tilde\Lambda.
$$

By composing (2.15) with the diagonal injection $T^*_\Lambda\tilde{\Lambda} \to T^*_\Lambda\tilde{\Lambda} \underset{\Lambda}{\times} T^*_\Lambda\tilde{\Lambda}$, we obtain

$$(2.16) \qquad \omega^r_\Lambda : T^*_\Lambda\tilde{\Lambda} \to T^*_{\mathrm{rel}}T^*_\Lambda\tilde{\Lambda},$$

which defines the relative canonical 1-form of $T^*_\Lambda\tilde{\Lambda}$. In case $\Lambda = \Lambda_0$, ω_Λ is expressed by coordinates as

$$(2.17) \qquad \omega^r_\Lambda = \sum_{j=1}^{d} z^*_j dz_j .$$

See § 2.9.4 of Y. Laurent [4] for details about ω^r_Λ.

The relative canonical 2-form Ω^r_Λ induces an isomorphism

$$(2.18) \qquad H^r_\Lambda : T^*_{\mathrm{rel}}T^*_\Lambda\tilde{\Lambda} \to T_{\mathrm{rel}}T^*_\Lambda\tilde{\Lambda} .$$

For a function defined on an open subset U of $T^*_\Lambda\tilde{\Lambda}$, we put

$$(2.19) \qquad H^r_f = H^r_\Lambda(\overline{df}) .$$

Here $\overline{df}$ is the image of df by $T^*T^*_\Lambda\tilde{\Lambda} \to T^*_{\mathrm{rel}}T^*_\Lambda\tilde{\Lambda}$. H^r_f defines a section of $T_{\mathrm{rel}}T^*_\Lambda\tilde{\Lambda}$ and gives a vector field on U. H^r_f is called the relative Hamiltonian vector field of f. In case $\Lambda = \Lambda_0$, it is written as

$$(2.20) \qquad H^r_f = \sum_{j=1}^{d} (\partial f/\partial z^*_j . \partial/\partial z_j - \partial f/\partial z_j . \partial/\partial z^*_j) .$$

Let M be a real analytic manifold with its complexification X and Σ be a regular involutory submanifold of $\dot{T}^*_M X$ with its complexification Λ. $\tilde{\Sigma}$ is the union of all bicharacteristics of Λ that pass through Σ and called a partial complexification of Σ. $T^*_\Lambda\tilde{\Lambda}$ is a natural complexification of $T^*_\Sigma\tilde{\Sigma}$. Thus real bisymplectic structure is induced on $T^*_\Sigma\tilde{\Sigma}$ from $T^*_\Lambda\tilde{\Lambda}$. We can define the relative Hamiltonian vector field for a function defined on an open set of $T^*_\Sigma\tilde{\Sigma}$.

Hereafter we restrict ourselves to the case $\Lambda = \Lambda_0$.

$T^*_\Lambda\tilde{\Lambda}$ has the following two $\mathbf{C}^\times$ actions. Let $\lambda \in \mathbf{C}^\times$.

$$(2.21) \qquad (w,z,\theta,z^*) \to (w,z,\lambda\theta,\lambda z^*) .$$

$$(2.22) \qquad (w,z,\theta,z^*) \to (w,z,\theta,\lambda z^*) .$$

Suppose that the map

$$\varphi : U \to U'$$

between open sets U and U′ of $\dot{T}^*_\Lambda\tilde{\Lambda}$ reserves the two $\mathbf{C}^\times$ actions and satisfies

$$(2.23) \qquad \varphi^* : \Omega_\Lambda = \Omega_\Lambda .$$

Then

$$\varphi^* : T^*T^*_\Lambda\tilde{\Lambda} \underset{T^*_\Lambda\Lambda}{\times} U' \to T^*T^*_\Lambda\tilde{\Lambda} \underset{T^*_\Lambda\tilde{\Lambda}}{\times} U$$

induces the morphism

$$\varphi^* : T^*_{\mathrm{rel}}T^*_\Lambda\tilde{\Lambda} \underset{T^*_\Lambda\tilde{\Lambda}}{\times} U' \to T^*_{\mathrm{rel}}T^*_\Lambda\tilde{\Lambda} \underset{T^*_\Lambda\tilde{\Lambda}}{\times} U .$$

Moreover we assume

$$(2.24) \qquad \varphi^*\Omega^r_\Lambda = \Omega^r_\Lambda .$$

Then φ is called a homogeneous bicanonical transformation.

Associated with φ, we can construct a ring isomorphism

$$(2.25) \qquad \Phi : \mathscr{E}^{2,(r,1)}_\Lambda|_U \to \mathscr{E}^{2,(r,1)}_\Lambda|_{U'},$$

and

$$(2.26) \qquad \Phi : \mathscr{E}^{2,\infty}_\Lambda|_U \to \mathscr{E}^{2,\infty}_\Lambda|_{U'},$$

which is called the quantized homogeneous bicanonical transformation associated with φ.

Φ satisfies the following properties:

$$(2.27) \qquad \Phi(\mathscr{E}^{2,(r,1)}_\Lambda[i,j]) \subset \mathscr{E}^{2,(r,1)}_\Lambda[i,j] .$$

$$(2.28) \qquad \sigma^{(r,1)}_\Lambda(\Phi(P)) = \sigma^{(r,1)}_\Lambda(P) \circ \varphi^{-1}$$

for any $P \in \mathscr{E}^{2,(r,1)}_\Lambda$.

3. Announcement of the main theorem.

3.1. Reduction to microlocal canonical form.

Let P be a microdifferential operator defined in a neighborhood of $\rho_0 \in \sqrt{-1}\,\mathring{T}^*\mathbf{R}^n$ which satisfies the conditions (1,1), (1.2), (1.3) and (1.4). As mentioned in the introduction, by Laubin [5], [6] and Melrose-Uhlmann [8] we can find a real quantized contact transformation that transforms the equation $Pu = 0$ into

(3.1) $$P_0 u = \{D_1^2 - \sum_{i,j=2}^{d} A^{ij}(x,D)D_iD_j + (\text{lower order})\}\, u = 0$$

defined in a neighborhood of $\rho_0 = (0, \sqrt{-1}\, dx_n) \in \sqrt{-1}\,T^*\mathbf{R}^n$.

Here we assume that

(3.2) $$\{A^{ij}\} \text{ are of order } 0$$

and

(3.3) $$(\sigma(A^{ij}))_{2\leqslant i,j\leqslant d} \text{ is positive definite.}$$

We study the 2-microlocal structure of the solutions of (3.1). We take a regular involutory submanifold Σ of $\sqrt{-1}\,\mathring{T}^*\mathbf{R}^n$ as

(3.4) $$\Sigma = \{(x, \sqrt{-1}\xi\, dx);\, \xi_1 = \cdots = \xi_d = 0\}$$

and a complexification of Σ in $\mathring{T}^*\mathbf{C}^n$ as

(3.5) $$\Lambda = \{(z,\zeta\, dz);\, \zeta_1 = \cdots = \zeta_d = 0\}.$$

We assume by (1.4)

(3.6) P_0 has regular singularities along Λ in the sense of Kashiwara-Oshima [3].

We regard Σ as a submanifold of Λ and define $\tilde{\Sigma}$ as the union of all bicharacteristics of Λ that pass through Σ.

We take a coordinate of $T^*_{\Sigma}\tilde{\Sigma}$ as $(x; \sqrt{-1}\xi''\, dx''; \sqrt{-1}\,x'^*\, dx')$ with $(x, \sqrt{-1}\xi''\, dx'') \in \Sigma$ and $x'^* = (x_1^*, \ldots, x_d^*) \in \mathbf{R}^d$.

For a function f defined on a neighborhood of a point of $T^*_{\tilde\Sigma}\tilde{\Sigma}$, we define a relative Hamilton vector field of f by

$$(3.7) \qquad H^r_f = \sum_{j=1}^{d} (\partial f/\partial x_j^* . \partial/\partial x_j - \partial f/\partial x_j . \partial/\partial x_j^*)$$

which is canonically defined from the bisymplectic structure of $T^*_{\tilde\Sigma}\tilde{\Sigma}$. See § 3.2 for H^r_f.

On $T^*_{\tilde\Sigma}\tilde{\Sigma}\setminus\Sigma$, the sheaf $\mathscr{C}^2_\Sigma$ of 2-microfunctions is defined. The sheaf $\mathscr{C}^2_\Sigma$ is used to study properties of microfunctions defined on Σ more precisely. Explicitly, there exists the sheaf $\mathscr{B}^2_\Sigma$ of 2-hyperfunctions on Σ, which relates $\mathscr{C}^2_\Sigma$ and $\mathscr{C}_{\mathbf{R}^n}|_\Sigma$ by the exact sequences ($\pi_\Sigma : T^*_{\tilde\Sigma}\tilde{\Sigma}\setminus\Sigma \to \Sigma$)

$$(3.8) \qquad 0 \to \mathscr{C}_\Sigma|_\Sigma \to \mathscr{B}^2_\Sigma \to \pi_{\Sigma*}\mathscr{C}^2_\Sigma \to 0$$

and

$$(3.9) \qquad 0 \to \mathscr{C}_{\mathbf{R}^n}|_\Sigma \to \mathscr{B}^2_\Sigma .$$

Here $\mathscr{C}_\Sigma$ denotes the sheaf of microfunctions with holomorphic parameters $(z_1, \ldots, z_d)$.

Moreover there exists canonical spectral map

$$(3.10) \qquad sp^2_\Sigma : \pi_\Lambda^{-1}\mathscr{B}^2_\Sigma \to \mathscr{C}^2_\Sigma .$$

We denote for $u \in \mathscr{B}^2_\Sigma$.

$$(3.11) \qquad SS^2_\Sigma(u) = \operatorname{supp}(Sp^2_\Sigma(u)),$$

which is called 2-singular spectrum of u along Σ.

For details about 2-microfunctions, see Kashiwara-Laurent [2].

Now we announce our main theorem.

THEOREM 3.1. — *For a microfunction solution* u *of* (3.1) *under the assumptions* (3.2), (3.3) *and* (3.6),

$$(3.12) \qquad SS^2_\Sigma(u) \subset \{(x; \sqrt{-1}\xi''; \sqrt{-1}x'^*) \in \mathring{T}^*_{\tilde\Sigma}\tilde{\Sigma}; f_0 = 0\}.$$

Moreover,

$$(3.13) \qquad SS^2_\Sigma(u) \textit{ is invariant under } H^r_{f_0}.$$

Here $f_0 = \sigma_\Lambda^{(\infty,1)}(P_0) = x_1^{*2} - \sum_{i,j=2}^{d} \sigma(A^{ij})(x, \xi'=0, \xi'')x_i^* x_j^*$.

Proof of theorem 3.1 will appear in § 4.

We define the propagation cone for 2-microlocal singular support by

$$(3.14)\quad \tilde{\Gamma}_{+} = \pi_{\Sigma}(\{\exp sH^{r}_{f_0})(0;\sqrt{-1}\,dx_n;\sqrt{-1}\,x'^{*}\,dx');x_1^{*} > 0,$$

$$f_0(0;\sqrt{-1}\,dx_n;\sqrt{-1}\,x'^{*}\,dx')=0,\ s\geqslant 0\}).$$

Here $\tau = (0;\sqrt{-1}\,dx_n;\sqrt{-1}\,x'^{*}\,dx')$ denotes a point of $\pi_{\Sigma}^{-1}(0,\sqrt{-1}\,dx_n)$ and $\exp(s\Theta)(\tau)$ denotes the exponential map for a vector field Θ starting from τ.

We give a theorem of microlocal Holmgren type for (3.1).

THEOREM 3.2. — *There exists a neighborhood Ω of $\rho_0 = (0,\sqrt{-1}\,dx_n)$ in $\sqrt{-1}\mathrm{T}^{*}\mathbf{R}^n$ such that for any microfunction solution u of* (3.1),

$$(3.15)\qquad \Omega \cap \mathrm{SS}(u) \cap (\tilde{\Gamma}_{+}\setminus\{\rho_0\}) = \varnothing$$

implies

$$(3.16)\qquad \mathrm{SS}(u) \not\ni \rho_0 .$$

Theorem 3.2 is an easy consequence of theorem 3.1 when we consult the exact sequences (3.8) and (3.9).

3.2. Theorems in invariant form.

We consider a microdifferential equation $\mathrm{P}u = 0$ defined in a neighborhood of $\rho_0 \in \sqrt{-1}\mathrm{T}^{*}\mathbf{R}^n$ that satisfies the conditions (1.1), (1.2), (1.3) and (1.4). We give theorems about P that can be reduced to theorem 3.1 and theorem 3.2.

Let

$$(3.17)\qquad \Sigma = \{\rho \in \sqrt{-1}\mathrm{T}^{*}\mathbf{R}^n;\, p(\rho)=0,\, dp(\rho)=0\}$$

and Λ be a complexification of Σ in $\mathrm{T}^{*}\mathbf{C}^n$. $\tilde{\Sigma}$ is the union of all bicharacteristic leaves of Λ issued from Σ. Then we have an isomorphism

$$(3.18)\qquad \mathrm{H}_{\Sigma}:\ \mathrm{T}^{*}_{\Sigma}\tilde{\Sigma} \simeq \mathrm{T}_{\Sigma}(\sqrt{-1}\mathrm{T}^{*}\mathbf{R}^n)$$

through the Hamiltonian isomorphism $H : T^*T^*\mathbf{C}^n \simeq TT^*\mathbf{C}^n$. Take a point $\rho \in \Sigma$ and put for $\tau \in T^*_\Sigma\tilde{\Sigma}|_\rho$

(3.19) $$p_\Sigma(\tau) = <\mathrm{Hess}\,(p)(\rho).\mathrm{H}_\Sigma(\tau), \mathrm{H}_\Sigma(\tau)> .$$

We remark that p_Σ is well defined as a function on $T^*_\Sigma\tilde{\Sigma}$.

Σ [resp. $T^*_\Sigma\tilde{\Sigma}\setminus\Sigma$] is endowed with the sheaf $\mathscr{B}^2_\Sigma$ [resp. $\mathscr{C}^2_\Sigma$] of 2-hyperfunctions [resp. 2-microfunctions]. Moreover $\mathscr{B}^2_\Sigma$ and $\mathscr{C}^2_\Sigma$ satisfy the same properties listed in § 3.1.

Because

(3.20) $$p_\Sigma = \sigma^{(\infty,1)}_\Lambda(\mathrm{P}_0)$$

in case $\mathrm{P} = \mathrm{P}_0$, we have

THEOREM 3.3. — *Let u be a microfunction solution of* $\mathrm{P}u = 0$ *defined in a neighborhood of* ρ_0. *Then*

(3.21) $$\mathrm{SS}^2_\Sigma(u) \subset \{(\rho,\tau) \in T^*_\Sigma\tilde{\Sigma}\setminus\Sigma; p_\Sigma(\rho,\tau) = 0\}.$$

Moreover

(3.22) $\mathrm{SS}^2_\Sigma(u)$ *is invariant under* $\mathrm{H}^r(p_\Sigma)$. *Here* $\mathrm{H}^r(p_\Sigma)$ *is the relative Hamiltonian vector field of* p_Σ *defined in* § 3.2. (*See also Remark* 3.5 *below.*)

We set

(3.23) $$\Gamma_{\rho_0} = \{\tau \in T^*_\Sigma\tilde{\Sigma}\,|_{\rho_0}; p_\Sigma(\rho_0,\tau)=0,\ \tau\neq 0\}$$

which consists of two conic sets Γ_+ and Γ_- in $(T^*_\Sigma\tilde{\Sigma}\setminus\Sigma)|_{\rho_0}$. We define the propagation cone for 2-microlocal singular support by

(3.24) $$\tilde{\Gamma}_+ = \pi_\Sigma(\{\exp\,(s\mathrm{H}^r(p_\Sigma))(\rho_0,\tau); \tau \in \Gamma_+, s\geqslant 0\}).$$

Here $\exp\,(s\Theta)(\rho,\tau)$ denotes the flow of a vector field Θ starting from (ρ,τ) and $\pi_\Sigma : T^*_\Sigma\tilde{\Sigma}\setminus\Sigma \to \Sigma$.

THEOREM 3.4. — *There exists an open neighborhood* Ω *of* ρ_0 *in* $\sqrt{-1}T^*\mathbf{R}^n$ *such that for a microfunction solution of* $\mathrm{P}u = 0$

(3.25) $$\Omega \cap \mathrm{supp}\,(u) \cap (\tilde{\Gamma}_+\setminus\{\rho_0\}) = \varnothing$$

implies $\mathrm{supp}\,(u) \not\ni \rho_0$.

Remark 3.5. – The relative Hamiltonian vector fields are also constructed in the following way. We have the identification

$$T^*_\Sigma\tilde{\Sigma} \simeq \cup\, T^*T\Gamma \tag{3.26}$$

where the union in the right side is taken for all bicharacteristic leaves of Σ. Take any Γ and put

$$p_\Gamma = p_\Sigma|_{T^*\Gamma}. \tag{3.27}$$

We remark that for a function f defined in an open subset of $T^*_\Sigma\tilde{\Sigma}$, H'_f is tangent to $T^*\Gamma$ and

$$H'_f|_{T^*\Gamma} = H_\Gamma(d(f|_{T^*\Gamma})). \tag{3.28}$$

Here H_Γ is the Hamiltonian isomorphism $T^*T^*\Gamma \to TT^*\Gamma$. Thus we may say

$$SS^2_\Sigma(u) \text{ is invariant under } H_\Gamma\,(dp_\Gamma)$$

in theorem 3.3.

Remark 3.6. – We set

$$\Gamma'_{\rho_0} = \{\tau \in T^*_\Sigma\tilde{\Sigma}|_{\rho_0}; p_\Sigma(\rho_0,\tau) \geqslant 0\}, \tag{3.29}$$

which consists of two solids Γ'_+ and Γ'_- in $T^*_\Sigma\tilde{\Sigma}_\rho|_{\rho 0.}$ P. Laubin [5] showed that in the situation of theorem 3.4,

$$\Omega \cap \operatorname{supp}(u) \cap (\tilde{\Gamma}'_+ \setminus \{\rho_0\}) = \varnothing \tag{3.30}$$

implies $\operatorname{supp}(u) \not\ni \rho_0$. Here

$$\tilde{\Gamma}'_+ = \pi_\Sigma(\{\exp(sH'_{p_\Sigma})(\rho_0,\tau); \tau \in \Gamma'_+, s \geqslant 0\}). \tag{3.31}$$

We remark that $\tilde{\Gamma}_+$ in (3.24) is the boundary of Γ'_+.

Remark 3.7. – Using Microlocal Study of Sheaves [14] developed by M. Kashiwara and P. Schapira, we can prove the same results of § 3 without the assumption (1.4). See N. Tose [13], where systems of microdifferential equations with conical refraction are treated.

4. Proof of the main theorem [proof of theorem 3.1].

We take a coordinate of $T_\Lambda^*\tilde{\Lambda}$ as $(z;\zeta''\,dz'';z'^*\,dz')$ where $(z,\zeta''\,dz'')$ denotes a point of Λ and $z'^*\in\mathbf{C}^d$.

We regard P_0 as a 2-microdifferential operator defined in a neighborhood of $\tau_0\in\pi_\Lambda^{-1}(\rho_0)$ in $T_\Lambda^*\tilde{\Lambda}$ where $\sigma_\Lambda^{(\infty,1)}(P_0)(\tau_0)=0$.

We may assume $z_d^*\neq 0$ at τ_0. Then 2-microlocally it is enough to consider the equation

$$P_1u = D_d^{-1}P_0u = 0. \tag{4.1}$$

Here we remark that

$$P_1\in\mathscr{E}_\Lambda^{2,(\infty,1)}[1,1]. \tag{4.2}$$

First we construct a homogeneous bicanonical transformation φ, which transforms $\sigma_\Lambda^{(\infty,1)}(P_1)$ into z_1^*.

We put $q=(z'',\zeta''\,dz'')$. Then $(z',z'^*;q)$ denotes a point $(z',z'';\zeta''\,dz'';z'^*\,dz')\in T_\Lambda^*\tilde{\Lambda}$. We take $\tilde{z}'\in\mathbf{C}^d$ and its dual variables $\tilde{z}'^*\in\mathbf{C}^d$. We define locally in $T^*\mathbf{C}^d\times T_\Lambda^*\tilde{\Lambda}$

$$\begin{aligned} Z = \{(\tilde{z}',\tilde{z}'^*;z',z'^*,q);\ & z_1=\tilde{z}_1=0,\ z_j=\tilde{z}_j\ (2\leqslant j\leqslant d),\\ & \tilde{z}_1^*=f(z',z'^*,q),\ z_j^*=-\tilde{z}_j^*\ (2\leqslant j\leqslant d)\}. \end{aligned} \tag{4.3}$$

Here $f=\sigma_\Lambda^{(\infty,1)}(P_1)$.

We integrate Z along the integral curves of relative Hamilton vector field.

$$H_F^r = \partial/\partial\tilde{z}_1 - H_f^r\quad (F=\tilde{z}_1^*-f). \tag{4.4}$$

Then we obtain a locally defined $2n$ dimensional submanifold $\tilde{Z}$ of $T^*\mathbf{C}^d\times T_\Lambda^*\tilde{\Lambda}$. It is easy to show (z',z'^*,q) is a coordinate of $\tilde{Z}$. When we write

$$\tilde{Z} = \{\tilde{z}'=\tilde{z}'(z',z'^*,q),\ \tilde{z}'^*=\tilde{z}'^*(z',z'^*,q)\}, \tag{4.5}$$

$$(z',z'^*,q)\xrightarrow{\ \varphi\ }(\tilde{z}'(z',z'^*,q),\ \tilde{z}'^*(z',z'^*,q),q) \tag{4.6}$$

defines a homogeneous bicanonical transformation. Moreover

$$\tilde{z}_1^*(z',z'^*,q) = f. \tag{4.7}$$

We quantized the transformation φ obtained above. Then the equation (4.1) is transformed into

$$P_2 u = 0 \tag{4.8}$$

defined in a neighborhood of $\tau_1 = (0;\sqrt{-1}\,dx_n;\sqrt{-1}\,dx_d) \in T^*_\Lambda\tilde{\Lambda}$, where

$$\sigma_\Lambda^{(\infty,1)}(P_2) = z_1^* \tag{4.9}$$

and

$$P_2 \in \mathscr{E}_\Lambda^{2,(\infty,1)}[1,1]. \tag{4.10}$$

Using the preparation theorem for $\mathscr{E}_\Lambda^{2,(\infty,1)}$, P_2 is written as

$$P_2 = Q(D_1 + B(x,D')) \tag{4.11}$$

with $D' = (D_2,\ldots,D_n)$. Here

(4.12) $\quad Q$ is inversible at τ_1

and

$$S(B) = \{(j,i);B_{ij} \not\equiv 0\} \subset \{(j,i);j \leqslant 1, i \geqslant j, i \geqslant 2j\}. \tag{4.13}$$

The right side of (4.13) is drawn in figure 4.1.

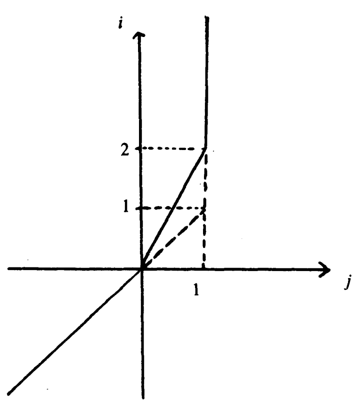

Figure 4.1.

By (4.11) and (4.12), we may assume from the beginning

(4.14) $$P_2 = D_1 + B(x,D')$$

under the assumption (4.13).

We find $R(x,D') \in \mathscr{E}_\Lambda^{2,\infty}$ satisfying

(4.15) $$P_2 R(x,D') = R(x,D')D_1$$

and

(4.16) $$R \text{ is invertible at } \tau_1 .$$

In the same way as Sato-Kawashiwara [9], we define formally $R = \sum_{l \geqslant 0} R^{(l)}(x,D')$ by finding $\{R^{(l)}\}$ recursively as follows:

(4.17) $$R^{(0)} \equiv 1 .$$

(4.18) $\partial/\partial z_1 . R^{(l)}(z,D') = B(z,D')R^{(l-1)}(z,D')(l \geqslant 1)$.

(4.19) $R^{(l)}(z,D')|_{z_1=1} = 0 \ (l \geqslant 1)$.

$\{R^{(l)}\}$ are given in an explicit manner by

(4.20) $$R^{(l)} = \int_0^{x_1} B(s_l, \hat{x}, D') \int_0^{s_l} B(s_{l-1}, \hat{x}, D') \dots \dots \int_0^{s_2} B(s_1, \hat{x}, D')\, ds_1 \dots ds_l$$

with $\hat{x} = (x_2, \dots, x_n)$.

Since the coefficients of $B(x,D')$ are holomorphic, we may assume $R^{(l)}$ is given by

(4.21) $$\int_{V_l} \dots \int B(s_l, \hat{x}, D') \dots B(s_1, \hat{x}, D')\, ds_1 \dots ds_l .$$

Here V_l denotes a real l-dimensional simplex whose volume is $|x_1|^l/l!$. We remark that

(4.22) $S(R^{(l)}) = \{(j,i); R_{ij}^{(l)} \not\equiv 0\} \subset \{(j,i); j \leqslant l, i \geqslant j, i \geqslant 2j\}$.

We put for $l \in \mathbf{N}$

(4.23) $$\mathscr{E}_\Lambda\langle l\rangle = \{P \in \mathscr{E}_\Lambda^{2,(2,1)}[2l,l]; P_{ij} \equiv 0 \quad i<j\} .$$

Then we have for $l,\ l' \in \mathbf{N}$

$$(4.24)\qquad \mathscr{E}_\Lambda\langle l\rangle \mathscr{E}_\Lambda\langle l'\rangle \subset \mathscr{E}_\Lambda\langle l+l'\rangle .$$

We remark that

$$(4.25)\qquad R^{(l)} \in \mathscr{E}_\Lambda\langle l\rangle \quad (l \geqslant 1)$$

and

$$(4.26)\qquad B \in \mathscr{E}_\Lambda\langle 1\rangle .$$

To prove the convergence for $\Sigma_l\ R^{(l)}$ in $\mathscr{E}_\Lambda^{2,\infty}$, we define the Formal Norm for $\mathscr{E}_\Lambda\langle l\rangle$.

DEFINITION 4.1. – *Let* U *be an open subset of* $T^*_\Lambda\tilde{\Lambda}$ *and* $P = \sum_{i,j} P_{ij} \in \mathscr{E}_\Lambda\langle l\rangle(U)$. *For a compact subset* K *of* U, *we define the Formal Norm for* P *on* K *by*

$$(4.27)\qquad N_K^{(l)}(P,s,t) = \sum_{i',j',\alpha,\beta} \frac{2(2(n+d))^{j'}(-j')!}{(-j'+|\alpha|)!(-j'+|\beta|)!} \sup_K |P^{\alpha,\beta}_{2l+i',l+j'}|\, s^{-2i'+|\alpha_1|+|\beta_1|}t^{-2(j'-2i')+|\alpha_2|+|\beta_2|}$$
$$= \sum_{i,k,\alpha,\beta} \frac{2(2(n+d))^{j+k-l}(l-i-k)!}{(l-i-k+|\alpha|)!(l-i-k+|\beta|)!} \sup_K |P^{\alpha,\beta}_{i,i+k}|\, s^{4l-2i+|\alpha_1|+|\beta_1|}t^{-2l-2k+|\alpha_2|+|\beta_2|}$$

where

$$P^{\alpha,\beta}_{ij} = (\partial/\partial z'^*)^{\alpha_1}(\partial/\partial\zeta'')^{\alpha_2}(\partial/\partial z)^{\beta} P_{ij}$$

and

$$\beta = (\beta_1,\beta_2) \in \mathbf{Z}^d \times \mathbf{Z}^{n-d}.$$

Remark 4.2. – For $P_1 \in \mathscr{E}_\Lambda\langle l\rangle$ and $P_2 \in \mathscr{E}_\Lambda\langle l'\rangle$,

$$(4.28)\qquad N_K^{(l+l')}(P_1P_2,s,t) \ll N_K^{(l)}(P_2,s,t)N_K^{(l')}(P_1,s,t).$$

We can prove the formula above by modifying theorem 2.4.9 of Y. Laurent [4].

Remark 4.3. – For $P \in \mathscr{E}_\Lambda\langle l\rangle(U)$ and a compact subset K of U, there exists a positive number C such that

$$N_K^{(l)}(P,s,t) \text{ converges on } \left\{(s,t);\, 0<|t|<\frac{1}{C}|s|^2<\left(\frac{1}{C}\right)^2\right\}.$$

Moreover $N_K^{(l)}(P_1,s,t)$ is bounded on

$$\{(s,t);|t/s^2| = \eta,|s|<\eta\}$$

for sufficiently small $\eta > 0$.

Now we go back to prove the convergence of $\Sigma_l R^{(l)}$.

We put

$$R = \Sigma R_{i,i+k} \text{ and } R^{(l)} = \Sigma R^{(l)}_{i,j+k}. \tag{4.29}$$

Then we have the estimates on a compact neighborhood K of τ_1

$$\sup_K |R^{(l)}_{i,i+k}| \leqslant B^l(l-i-k)!/2.2(n+d))^{i+k-l}(t^2/s^4)^l s^{2i}t^{2k} \tag{4.30}$$

with

$$B = \sup_K |z_1|.N_K^{(1)}(A,s,t). \tag{4.31}$$

We remark that

$$\{(i,k);R_{i,i+k} \neq 0\} \subset \left\{(i,k);k\leqslant,0,k\leqslant\frac{1}{2}i\right\}. \tag{4.32}$$

(I) *Estimates for* $R_{i,i+k}$ *in case* $i, k \leqslant 0$.

When $i, k \leqslant 0$, we have on K

$$\begin{aligned} |R_{i,i+k}| &\leqslant (s^2/2(n+d))^i(t^2/2(n+d))^k \\ &\qquad \sum_l (t^2/s^4)^l(2(n+d))^{-l}B^l(l-i-k)!/l! \\ &\leqslant (s^2/2(n+d))^i(t^2/2(n+d))^k \\ &\qquad \sum_{l\geqslant 0} (t^2B/(2(n+d)s^4)^l.2^{l-i-k}(-i)!(-k)! \\ &= (4(n+d)/s^2)^{-i}(4(n+d)/t^2)^{-k} \\ &\qquad \sum_{l\geqslant 0} (t^2B/(2(n+d)s^4)^l(-i)!(-k)! \end{aligned} \tag{4.33}$$

When we fix (s,t) so that $N_K^{(1)}(A,s,t)$ converges and take K small enough, we can take a positive C_K satisfying

$$\sup_K |Ri,i+k| \leqslant C_K^{-i-k}(-i)!(-k)!. \tag{4.34}$$

(II) *Estimates for* $R_{i,i+k}$ *in case* $k \leqslant 0$, $i > 0$.

We have on a compact neighborhood K of τ_1

$$(4.35)\quad |R_{i,i+k}| \leqslant (s^2/2(n+d))^i 2(2n+d)/t^2)^{-k} \sum_{l\geqslant 0} (t^2/s^4)^l (2(n+d))^{-l} B^l (l-i-k)!/l!$$
$$\leqslant (s^2/2(n+d))^i (2(n+d))/t^2)^{-k} \sum_{l\geqslant 0} (t^2/s^4)^l (2(n+d))^{-l} . B^l . 2^{l-i-k}(-k)!/i!$$
$$= (s^2/4(n+d))^i (4(n+d)/t^2)^{-k} \sum_{l} (t^2 B/s^4(n+d))^l (-k)!/i!.$$

We move (s,t) on $\{(s,t); t=\eta^2, s\leqslant\eta\}$ for small η and take K small enough so that

$$(4.36)\qquad \eta^2 B/(n+d) \leqslant 1/2.$$

Here we remark that B is uniformly bounded on $\{(s,t); t=\eta s^2, s\leqslant\eta\}$ if we take η small enough.

Thus we have a positive $C_{\varepsilon,K}$ for any positive ε such that

$$(4.37)\qquad \sup_K |R_{i,i+k}| \leqslant \varepsilon^i C_{\varepsilon,K}^{-k}(-k)!/i!.$$

By the estimates (I) and (II), we conclude that

$$(4.38)\qquad R(x,D') \in \mathscr{E}_\Lambda^{2,\infty}(U)$$

for some neighborhood U of τ_1. Thus we find $R(x,D') \in \mathscr{E}_\Lambda^{2,\infty}(U)$ satisfiying

$$(4.39)\qquad P_2(x,D)R(x,D') = R(x,D')D_1.$$

We prove that $R(x,D')$ is inversible. But we can verify it by applying the same argument of theorem 5.2.1 in chapter 2 of Sato-Kawai-Kashiwara [9].

To sum up, we have

THEOREM 4.4. — *Let P_2 be a section of $\mathscr{E}_\Lambda^{2,(\infty,1)}[1,1]$ in (4.14) defined in a neighborhood of $\tau_1 = (0;\sqrt{-1}\,dx_n;\sqrt{-1}\,dx_d) \in T_\Lambda^*\tilde{\Lambda}$, with $\sigma_\Lambda^{(\infty,1)}(P_2) = z_1^*$. Then we can find an inversible section $R(x,D')$ of $\mathscr{E}_\Lambda^{2,\infty}$ defined in a neighborhood of τ_1 satisfying*

$$(4.40)\qquad P_2(x,D)R(x,D') = R(x,D')D_1.$$

By theorem 4.4 above, we can prove

THEOREM 4.5. — *Let* P_2 *be a 2-microdifferential operator in theorem* 4.4 *and* u *be a section of* $\mathscr{C}^2_\Lambda$ *defined in a neighborhood of* τ_1. *Then*

$$\text{(4.41)} \quad \operatorname{supp} u \subset \{(x;\sqrt{-1}\,\xi''\,dx'';\sqrt{-1}\,x'^*\,dx');x_1^*=0\}.$$

Moreover $\operatorname{supp} u$ *is invariant under* $\partial/\partial x_1$.

Proof. — It is an easy consequence of theorem 4.4 and de Rham's theorem for $C^2_{\Sigma'}$ (q.e.d.).

Associated with

$$\text{(4.42)} \qquad \varphi_R = \varphi|_{T^*_\Sigma\tilde{\Sigma}} : U \cap T^*_\Sigma\tilde{\Sigma} \to U' \cap T^*_\Sigma\tilde{\Sigma},$$

we can construct an isomorphism

$$\text{(4.43)} \qquad \Phi_R : \mathscr{C}^2_\Sigma \to \varphi_R^{-1}\mathscr{C}^2_\Sigma$$

by § 3.3.4 of Kashiwara-Laurent [2]. Moreover Φ_R is compatible with $\mathscr{E}^{2,\infty}_\Lambda$-module structure of $\mathscr{C}^2_\Lambda$.

Hence we can prove theorem 3.1 by theorem 4.5.

BIBLIOGRAPHY

[1] R. COURANT, D. HILBERT, *Methods of Mathematical Physics,* vol. II, Interscience, 1962.

[2] M. KASHIWARA, Y. LAURENT, Théorèmes d'annulation et deuxième microlocalisation, *Prepublications d'Orsay*, Univ. Paris-Sud, 1983.

[3] M. KASHIWARA, T. OSHIMA, Systems of differential equations with regular singularities and their boundary value problems, *Ann. of Math.*, 106 (1977), 145-200.

[4] Y. LAURENT, *Théorie de la deuxième microlocalisation : opérateurs 2-micro-différentiels*, Thesis presented to Univ. Paris-Sud, Centre d'Orsay, 1982; published by Birkhauser as Progress in Math. No 53.

[5] P. LAUBIN, Thesis presented to Univ. Liège, 1983.

[6] P. LAUBIN, Réfraction conique et propagation des singularités analytiques, *J. Math. pures et appl.*, 63 (1984), 149-168. (See also a note in Sem. de Vaillant, Univ. Paris VI, 1982.)

[7] D. LUDWIG, Conical refraction in crystal optics and hydromagnetics, *Comm. in Pure and Appl. Math.*, 14 (1961), 113-124.

[8] R. MELROSE, G. UHLMANN, Microlocal structure of involutive conical refraction, *Duke Math. J.*, 46 (1979), 571-582.

[9] M. Sato, T. Kawai, M. Kashiwara, Microfunctions and Pseudodifferential Equations, *Lecture Note in Math.*, No 287, Springer, (1973), 265-529.

[10] P. Schapira, Microdifferential systems in the complex domain, *Grundlehren der Math.*, Springer, (1985).

[11] N. Tose, On a class of microdifferential equations with involutory double characteristics, to appear in *J. of Fac. Science, Univ. Tokyo*, 33-3 (1986), 619-634.

[12] N. Tose, The 2-Microlocal Canonical Form for a Class of Microdifferential Equations and Propagation of Singularities, to appear in *Publ. of RIMS, Kyoto Univ.*, 23-1 (1987).

[13] N. Tose, On a class of 2-microhyperbolic systems I and II, preprints.

[14] M. Kashiwara, P. Shapira, Microlocal Study of Sheaves, *Astérisque*, 128 (1985).

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