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DEFORMATION OF HOLOMORPHIC MAPS ONTO FANO MANIFOLDS OF SECOND AND FOURTH BETTI NUMBERS 1

by Jun-Muk HWANG

Abstract. — Let X be a Fano manifold with $b_2 = 1$ different from the projective space such that any two surfaces in X have proportional fundamental classes in $H_4(X, \mathbf{C})$. Let $f : Y \to X$ be a surjective holomorphic map from a projective variety Y. We show that all deformations of f with Y and X fixed, come from automorphisms of X. The proof is obtained by studying the geometry of the integral varieties of the multi-valued foliation defined by the variety of minimal rational tangents of X.

Résumé. — Soit X une variété de Fano avec $b_2 = 1$ différente de l'espace projectif et telle que tout couple de surfaces dans X ont des classes fondamentales dans $H_4(X, \mathbf{C})$ proportionnelles. Soit $f : Y \to X$ une application surjective d'une variété projective Y dans X. Nous montrons que toute déformation de f de Y dans X (fixés), provient d'automorphismes de X. La preuve est obtenue en étudiant la géométrie des variétés intégrales du feuilletage multi-valué défini par la variété des vecteurs tangents des courbes rationnelles minimales de X.

1. Introduction

Given two compact complex manifolds X, Y and a holomorphic map $f : Y \to X$, an obvious way to deform f as maps from Y to X is by the automorphisms of X. More precisely, let $\{g_t, t \in \mathbf{C}, |t| < 1\}$ be a 1-parameter family of automorphisms of X with $g_0 = \mathrm{Id}_X$. Then the family of holomorphic maps $\{g_t \circ f, |t| < 1\}$ defines a deformation of f. In this case, we say that the deformation of f comes from automorphisms of X. The main result of [5] says that when X is a projective manifold which is not uniruled, all deformations of a surjective holomorphic map $f : Y \to X$ come from automorphisms of the target up to an etale cover (see [5] for the precise

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statement). In particular, when X is a simply connected projective manifold which is not uniruled, all deformations of f come from automorphisms of X. For convenience, let us say that a compact complex manifold X has *the target rigidity property* if for any surjective holomorphic map $f : Y \to X$, all deformations of f come from automorphisms of X. Thus [5] says that a simply connected projective manifold which is not uniruled has the target rigidity property.

It is natural to ask what happens when X is a simply connected uniruled projective manifold, in particular, a Fano manifold. The projective space $\mathbf{P}_n$ does not have the target rigidity property. In fact, given an n-dimensional projective manifold Y embedded in some projective space $\mathbf{P}_N$, there are various ways to project Y to $\mathbf{P}_n \subset \mathbf{P}_N$ which give examples of deformations of surjective holomorphic maps to $\mathbf{P}_n$ which do not come from automorphisms of $\mathbf{P}_n$. However, $\mathbf{P}_n$ is somewhat exceptional in this regard. In fact, we expect the following.

Conjecture 1.1. — *Let X be a Fano manifold with second Betti number $b_2 = 1$, which is different from the projective space. Then X has the target rigidity property.*

The strongest evidence for Conjecture 1.1 is the following result of [7].

Theorem 1.2. — *[[7], Theorem 3] Let X be a Fano manifold with $b_2 = 1$, different from the projective space. If the variety of minimal rational tangents of X at a general point $x \in X$ is not a union of positive dimensional linear subspaces, then X has the target rigidity property.*

The technical assumption in Theorem 1.2 on the variety of minimal rational tangents holds for all examples of Fano manifolds with $b_2 = 1$ for which the variety of minimal rational tangents has been identified. In fact, the following stronger form of Conjecture 1.1 is expected.

Conjecture 1.3. — *[[7], p. 52, Conjecture] Let X be a Fano manifold with $b_2 = 1$, different from the projective space. Then the variety of minimal rational tangents of X at a general point $x \in X$ is not a union of positive dimensional linear subspaces.*

Since there seems to be no good approach to Conjecture 1.3 at the moment, it is worthwhile studying Conjecture 1.1 and trying to replace the technical assumption in Theorem 1.2 by less technical conditions. We will prove the following which is a result in this direction.

Theorem 1.4. — *Let X be an n-dimensional Fano manifold with $b_2 = 1$, different from the projective space. Suppose the fundamental classes in*

$H_4(X, \mathbf{C})$ of any two surfaces in X are proportional. For example, this is the case if X has $b_4 = 1$. Then X has the target rigidity property.

By Theorem 1.2, the proof of Theorem 1.4 is reduced to the case when the variety of minimal rational tangents at a general point of X is the union of linear subspaces of positive dimension. We will exploit the geometry of the integral varieties of the multi-valued foliation defined by the linear variety of minimal rational tangents to establish Theorem 1.4. In the course of our proof of Theorem 1.4, we will obtain the following partial verification of Conjecture 1.3 (*cf.* Corollary 2.3).

THEOREM 1.5. — *Let X be an n-dimensional Fano manifold with $b_2 = 1$. If the variety of minimal rational tangents at a general point $x \in X$ is a union of linear subspaces of dimension $\geqslant \frac{n-1}{2}$, then X is $\mathbf{P}_n$.*

Let us end the introduction with one application of Theorem 1.4.

COROLLARY 1.6. — *Let $Y \subset \mathbf{P}_{n+1}$ be a smooth hypersurface of dimension $n \geqslant 5$ and let X be a projective manifold different from $\mathbf{P}_n$. Given any surjective holomorphic map $f : Y \to X$, all deformations of f come from automorphisms of X.*

In fact, this is true for any smooth complete intersection Y of dimension $\geqslant 5$ in a projective space. Note that by Lefschetz, $b_1(Y) = 0$ and $b_2(Y) = b_4(Y) = 1$. Thus the Picard group of X is cyclic. If X is not Fano, the result follows from $b_1(X) = 0$ and [[5], Theorem 1.2]. Thus we may assume that X is a Fano manifold with $b_2 = 1$. Now $b_4(Y) = 1$ implies that any two surfaces in X have proportional fundamental classes in $H_4(X, \mathbf{C})$, so Theorem 1.2 can be applied.

Regarding Corollary 1.6, it is expected that when Y is a general hypersurface, no such map f exists. In fact, this was proved by Amerik [1] for a general hypersurface in $\mathbf{P}_4$.

2. Fano manifolds with linear variety of minimal rational tangents

For the back-ground material on minimal rational curves and the variety of minimal rational tangents, we refer the readers to [3]. Let X be a uniruled projective manifold and let $\mathcal{K}$ be a minimal dominating component of the space of rational curves on X. For a general point $x \in X$, let $\mathcal{K}_x$ be the subvariety of $\mathcal{K}$ consisting of members passing through x. The variety of minimal rational tangents at x (determined by $\mathcal{K}$) is the subvariety $\mathcal{C}_x$ of $\mathbf{P}T_x(X)$ defined as the closure of the set of the tangent directions to members of $\mathcal{K}_x$ smooth at x.

Throughout this section, we will make the following assumption.

Assumption. X is an n-dimensional Fano manifold of $b_2 = 1$, different from $\mathbf{P}_n$, and for some choice of $\mathcal{K}$, the variety of minimal rational tangents at a general point is the union of linear subspaces of dimension $p > 0$.

The condition $X \neq \mathbf{P}_n$ implies that $p < n - 1$ and the condition $b_2(X) = 1$ implies that $\mathcal{C}_x$ has at least two (in fact, three by [[4], Proposition 2]) irreducible components. In an analytic local neighborhood of x, each component of $\mathcal{C}_x$ defines a distribution. It is easy to see that this distribution is integrable and the leaf through x is an immersed $\mathbf{P}_{p+1}$. More precisely, we have the following.

Proposition 2.1. — *In the above setting, there exists a normal variety X' with a finite holomorphic map $\rho : X' \to X$ and a dense open subset U of X' equipped with a proper holomorphic map $\varphi : U \to T$ such that each fiber of φ is biholomorphic to $\mathbf{P}_{p+1}$ and each member of $\mathcal{K}_x$ for a general $x \in X$ is the image of a line in some fibers of φ. Moreover, for each $t \in T$, let $P_t := \rho(\varphi^{-1}(t))$ be the subvariety in X. Then P_t is an immersed submanifold with trivial normal bundle in X, $\rho|_{\varphi^{-1}(t)}$ is its normalization, and for two distinct points $t_1 \neq t_2 \in T$, the two subvarieties P_{t_1} and P_{t_2} are distinct.*

Here, when X is a complex manifold and $Z \subset X$ is a subvariety. We say that Z is an immersed submanifold if the normalization $\hat{Z}$ is smooth and the normalization map $\nu : \hat{Z} \to Z \subset X$ is an immersion. The normal bundle of Z means the vector bundle N_Z on $\hat{Z}$ defined as the quotient of $\nu^* T(X)$ by the image of $T(\hat{Z})$. If N_Z is a trivial bundle, we say that Z is an immersed submanifold with trivial normal bundle.

Proof. — The proof of the first sentence is given in [[2], Theorem 3.1] and the distinctness of P_{t_1} and P_{t_2} for $t_1 \neq t_2$ is obvious from the construction there. Let us check the triviality of the normal bundle. For a general point $x \in X$ and any member C of $\mathcal{K}$ through x, the splitting type of $T(X)$ on the normalization of C is $\mathcal{O}(2) \oplus \mathcal{O}(1)^p \oplus \mathcal{O}^{n-1-p}$. Let $C \subset P_t$. Then the splitting type of $T(\varphi^{-1}(t))$ on the normalization of C is $\mathcal{O}(2) \oplus \mathcal{O}(1)^p$. Thus the splitting type of the normal bundle of P_t on any such C through x is $\mathcal{O}^{n-p-1}$. It follows that the normal bundle is trivial by [[8], Theorem 3.2.1]. This shows P_t is an immersed submanifold with trivial normal bundle for a general $t \in T$. Hence we can say that it is true for each $t \in T$ by shrinking U and T. □

We have the following consequence.

PROPOSITION 2.2. — *In the above setting, two distinct irreducible components of $\mathcal{C}_x$ for a general point $x \in X$ are disjoint.*

Proof. — Suppose not. Then for a general point $x \in X$, there are two distinct points $t_1, t_2 \in T$ such that

$$x \in P_{t_1} \cap P_{t_2} \text{ and } \dim(P_{t_1} \cap P_{t_2}) \geqslant 1.$$

Choose a complete curve C, which is not necessarily a rational curve, such that

$$x \in C \subset P_{t_1} \cap P_{t_2}.$$

Let $\nu : \hat{C} \to C$ be the normalization and consider the diagram

$$\begin{array}{ccc} \hat{C} & \xrightarrow{\nu_1} & \varphi^{-1}(t_1) \\ \nu_2 \downarrow & & \downarrow \rho_1 \\ \varphi^{-1}(t_2) & \xrightarrow{\rho_2} & X \end{array}$$

where ν_1, ν_2 are the natural maps lifting ν and ρ_1, ρ_2 are the restrictions of ρ such that

$$\rho_1 \circ \nu_1 = \rho_2 \circ \nu_2 = \nu.$$

From Proposition 2.1, we can regard $\nu_i^* T(\varphi^{-1}(t_i))$ as a subbundle of $\nu^* T(X)$ via the immersion ρ_i for each $i = 1, 2$. Denote this subbundle by V_i for each $i = 1, 2$. Then

$$\nu^* T(X)/V_i \cong \mathcal{O}_{\hat{C}}^{n-p-1}$$

from the triviality of the normal bundle. Since V_1 and V_2 are two distinct subbundles of $\nu^* T(X)$, there exists a point $z \in \hat{C}$ such that the fibers $V_{1,z}$ and $V_{2,z}$ are different in $\nu^* T(X)_z$. Pick a point $y \neq z \in \hat{C}$ and let $\mathbf{m}_y$ be the maximal ideal. Since $V_1 = \nu_1^* T(\varphi^{-1}(t_1))$ and $\varphi^{-1}(t_1) \cong \mathbf{P}_{p+1}$, the natural homomorphism

$$H^0(\hat{C}, V_1 \otimes \mathbf{m}_y) \to V_{1,z}$$

is surjective. Then the image of V_1 in

$$\nu^* T(X)/V_2 \cong \mathcal{O}_{\hat{C}}^{n-p-1}$$

has sections vanishing at y but non-zero at z, a contradiction. $\square$

An immediate consequence is the following, which proves Theorem 1.5. Recall that the index of a Fano manifold Z is the largest integer i such that $-K_Z = iL$ for some line bundle L in $\operatorname{Pic} Z$.

Corollary 2.3. — *In the above setting,* $\dim \mathcal{C}_x = p < \frac{n-1}{2}$. *In particular, if Z is a Fano manifold of $b_2 = 1$ and of index $\geqslant \frac{n+3}{2}$, then for any choice of a minimal dominating component of the space of rational curves on Z, the variety of minimal rational tangents at a general point cannot be the union of linear subspaces, unless $Z \cong \mathbf{P}_n$.*

Proof. — If $\dim \mathcal{C}_x = p \geqslant \frac{n-1}{2}$, any two components of $\mathcal{C}_x$ must intersect in $\mathbf{P}T_x(X) \cong \mathbf{P}_{n-1}$, a contradiction to Proposition 2.2. Note that $p+2$ is the anti-canonical degree of the rational curves belonging to $\mathcal{K}$. Thus if the index of Z is $\geqslant \frac{n+3}{2}$, then $p \geqslant \frac{n-1}{2}$. □

3. Proof of Theorem 1.4

In this section, we will impose one more condition on X.

Assumption. X is as in Section 2 and any two surfaces in X have proportional fundamental classes in $H_4(X, \mathbf{C})$.

For example, this is the case if $b_4(X) = 1$, or X is the image of a holomorphic map from a projective manifold with $b_4 = 1$. The condition on $H_4(X, \mathbf{C})$ gives information about the map $\rho : X' \to X$ in Proposition 2.1, as follows.

Proposition 3.1. — *Let us use the notation of Proposition 2.1. Let $B \subset X$ be an irreducible hypersurface in X. Then there exists an irreducible component $D \subset X'$ of $\rho^{-1}(B)$ with the following properties.*

(i) D is dominant over T.

(ii) ρ is unramified at a general point of D.

(iii) $\rho|_D : D \to B$ has degree > 1.

Proof. — B is an ample hypersurface from $b_2(X) = 1$. Given B and a general point $x \in B$, there exists a member C of $\mathcal{K}$ intersecting B transversally at x (and possibly at other points) from the freeness of general members of $\mathcal{K}$. This implies that there is $t \in T$ with P_t intersecting B transversally at x. By Proposition 2.1, C is the birational image of a line C' in $\varphi^{-1}(t)$. This implies that ρ is unramified at $\rho^{-1}(x) \cap \varphi^{-1}(t)$ (e.g. *from the proof of [[6], Lemma 1]). Thus there exits D satisfying (i) and (ii). Suppose $\rho|_D$ is birational over B. We extend $\varphi|_D$ to a dominant rational map ϕ from D to some projective variety $\bar{T}$ compactifying T. Let $E \subset D$ be the indeterminacy locus of ϕ. Then $\rho(E)$ is a subvariety of codimension $\geqslant 2$ in B. Thus we can choose a general point $t \in T$ and a surface $S \subset P_t$ such that $S \cap B$ is a curve disjoint from $\rho(E)$. Since $\rho|_D : D \to B$ is finite* *and birational, we can write $(\rho|_D)^{-1}(S \cap B) = A \cup Q$ for an irreducible curve A contained in a fiber of φ and a finite set Q. By the choice of S, Q is disjoint from E. Thus we can choose a hypersurface in $\bar{T}$ such that its proper transform $F \subset D$ under $\phi|_D$ is disjoint from A and Q. It follows that $\rho(F) \subset B$ is disjoint from $S \cap B$, and consequently, from S. But by the assumption on $H_4(X, \mathbf{C})$, any subvariety of codimension 2 in X must have positive intersection with the surface S, a contradiction.* □

PROPOSITION 3.2. — *In the setting of Proposition 3.1, suppose we are given an irreducible hypersurface $B \subset X$, a general point $x \in B$ and an open neighborhood $W \subset X$ of x. Then there exists a point $y \in W$ and two distinct points $t_1, t_2 \in T$ with $y \in P_{t_1} \cap P_{t_2}$ and $T_y(P_{t_1}) \neq T_y(P_{t_2})$ such that the irreducible component of $W \cap P_{t_1}$ (resp. $W \cap P_{t_2}$) containing y intersects B transversally at some point of $B \cap W$.*

Proof. — Let $D \subset X'$ be as in Proposition 3.1. Let x_1, x_2 be two distinct points on $\rho^{-1}(x) \cap D$ from Proposition 3.1 (iii). We know that ρ are unramified at x_1 and x_2 from Proposition 3.1 (ii). There exist open neighborhoods $W_1 \subset U$ of x_1, $W_2 \subset U$ of x_2 and $W_0 \subset W$ of x with the following properties:

(1) $\rho(W_1) = \rho(W_2) = W_0$.
(2) $\rho|_{W_1}$ and $\rho|_{W_2}$ are biholomorphic.
(3) $W_1 \cap D$ and $W_2 \cap D$ are non-singular and transversal to fibers of φ.

There exists an open neighborhood $W_1' \subset W_1$ of x_1 (resp. $W_2' \subset W_2$ of x_2) such that for any $z \in W_1'$ (resp. $z \in W_2'$), $\varphi^{-1}(\varphi(z)) \cap W_1'$ (resp. $\varphi^{-1}(\varphi(z)) \cap W_2'$) is connected. Let y be a general point in $\rho(W_1') \cap \rho(W_2')$. Let $y_1 = W_1' \cap \rho^{-1}(y)$ and $y_2 = W_2' \cap \rho^{-1}(y)$. Then $t_1 := \varphi(y_1)$ and $t_2 := \varphi(y_2)$ give the desired two points. They must be distinct from the generality of y. □

We get the following consequence for subvarieties of the tangent bundle $T(X)$.

PROPOSITION 3.3. — *Let Y' be a projective subvariety in $T(X)$ dominant over X. Then the natural projection $Y' \to X$ is birational. In particular, Y' defines a section in $H^0(X, T(X))$.*

Proof. — Let Y be the normalization of Y' and $f : Y \to X$ be the induced finite holomorphic map. If f is birational, it is biholomorphic and consequently, the projection $Y' \to X$ is biholomorphic. Thus it suffices to show that f is birational.

Suppose that f is not birational. Let $R \subset Y$ be an irreducible ramification divisor of f and $B = f(R)$. Let $z \in R$ be a general point. Let r be the local sheeting number of f at z in the sense of [6]. We can choose a holomorphic coordinate neighborhood V of z with coordinates $(w_1, \ldots, w_n)$ at z and a holomorphic coordinate neighborhood W of $f(z)$ with coordinates $(z_1, \ldots, z_n)$ such that $f^{-1}(W) \subset V$ and f is given by

$$z_1 = w_1, \ \ldots, \ z_{n-1} = w_{n-1}, \ z_n = w_n^r.$$

By Proposition 3.2, there exists a point $x_o \in W \setminus B$, two distinct points $t_1, t_2 \in T$ and members C_1, C_2 of $\mathcal{K}$ such that

$$C_1 \subset P_{t_1}, \ C_2 \subset P_{t_2}, \ x_o \in C_1 \cap C_2$$

and the irreducible component of $C_1 \cap W$ (resp. $C_2 \cap W$) containing x_o intersects B transversally at some point of $B \cap W$. A direct computation in the coordinates of V and W, as in [[6], p. 636, Lemma 1], shows that for any irreducible complex analytic curve C on V intersecting B transversally, $f^{-1}(C) \cap W$ is irreducible. Thus there exists a unique irreducible component C_1' (resp. C_2') of $f^{-1}(C_1)$ (resp. $f^{-1}(C_2)$) intersecting V such that an irreducible component of $C_1' \cap V$ (resp. $C_2' \cap V$) contains $f^{-1}(x_o) \cap V$. In particular, $C_1' \cap C_2'$ contains the r distinct points $f^{-1}(x_o) \cap V$. Note that the holomorphic map $\nu : Y \to T(X)$ given by the normalization map of Y' induces a canonical section $\sigma \in H^0(Y, f^*T(X))$ such that

$$f_*(\sigma(y)) = \nu(y) \in T_{f(y)}(X).$$

From here, for the simplicity of the notation, we will assume that the immersed submanifolds $C_1, C_2, P_{t_1}, P_{t_2}$ of X are embedded submanifolds of X. It is easy to see that our argument below works verbatim without this assumption by lifting things to the normalizations of the immersed submanifolds. Consider the exact sequence

$$0 \longrightarrow H^0(C_1', f^*T(P_{t_1})) \longrightarrow H^0(C_1', f^*T(X)) \longrightarrow H^0(C_1', f^*N_{P_{t_1}})$$

where $N_{P_{t_1}}$ is the normal bundle of P_{t_1}. Since the normal bundle of P_{t_1} is trivial,

$$H^0(C_1', f^*N_{P_{t_1}}) = f^*H^0(C_1, N_{P_{t_1}}).$$

It follows that for any two distinct $y_1 \neq y_2 \in f^{-1}(x_o) \cap V$,

$$f_*(\sigma(y_1)) - f_*(\sigma(y_2)) \in T_{x_o}(P_{t_1}).$$

By the same argument, we get

$$f_*(\sigma(y_1)) - f_*(\sigma(y_2)) \in T_{x_o}(P_{t_2}).$$

However, by Proposition 2.2, we have $T_{x_o}(P_{t_1}) \cap T_{x_o}(P_{t_2}) = 0$. It follows that

$$f_*(\sigma(y_1)) = f_*(\sigma(y_2)) \text{ for any two } y_1, y_2 \in f^{-1}(x_o) \cap V.$$

But by the definition of the canonical section σ of $f^*T(X)$ on Y, $f_*(\sigma(y_1)) \neq f_*(\sigma(y_2))$ if $y_1 \neq y_2$. This is a contradiction. Thus $f: Y \to X$ must be birational. □

We are ready to prove Theorem 1.4.

Proof of Theorem 1.4. — Note that the Kodaira-Spencer class of a deformation of a holomorphic map $f: Y \to X$ is in $H^0(Y, f^*T(X))$. Thus to prove Theorem 1.4, it suffices to show that for any surjective holomorphic map $f: Y \to X$,

$$H^0(Y, f^*T(X)) = f^*H^0(X, T(X)).$$

Suppose $\sigma \in H^0(Y, f^*T(X))$. Then associating to $y \in Y$ the vector $f_*\sigma(y) \in T_{f(y)}(X)$, we get a holomorphic map $Y \to T(X)$ whose image Y' is a projective subvariety in $T(X)$. By Proposition 3.3, Y' is a section $v \in H^0(X, T(X))$. It follows that $\sigma = f^*v$. □

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