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Jonathan ARAZY, Miroslav ENGLIŠ & Wilhelm KAUP

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HOLOMORPHIC RETRACTIONS AND BOUNDARY BEREZIN TRANSFORMS

by Jonathan ARAZY, Miroslav ENGLIŠ & Wilhelm KAUP (*)

ABSTRACT. — In an earlier paper, the first two authors have shown that the convolution of a function f continuous on the closure of a Cartan domain and a K -invariant finite measure μ on that domain is again continuous on the closure, and, moreover, its restriction to any boundary face F depends only on the restriction of f to F and is equal to the convolution, in F , of the latter restriction with some measure μ_F on F uniquely determined by μ . In this article, we give an explicit formula for μ_F in terms of F , showing in particular that for measures μ corresponding to the Berezin transforms the measures μ_F again correspond to Berezin transforms, but with a shift in the value of the Wallach parameter. Finally, we also obtain a nice and simple description of the holomorphic retraction on these domains which arises as the boundary limit of geodesic symmetries.

RÉSUMÉ. — Dans un papier antérieur, les deux premiers co-auteurs ont démontré que la convolution d'une fonction f continue sur l'adhérence d'un domaine de Cartan avec une mesure finie μ K -invariante dans ce domaine est aussi continue sur l'adhérence. De plus, sa restriction à chaque face F de la frontière dépend uniquement de la restriction de f sur F et est égale à la convolution, dans F , de cette restriction-là, avec une certaine mesure μ_F sur F , déterminée uniquement par μ . Dans cet article nous donnons une formule explicite pour μ_F en termes de F , en montrant plus particulièrement que pour des mesures μ correspondant à des transformées de Berezin, les mesures μ_F correspondent à nouveau à des transformées de Berezin mais avec un décalage dans la valeur du paramètre de Wallach. Enfin, nous obtenons aussi une description simple et jolie d'une rétraction holomorphe sur ces domaines qui découle de la limite à la frontière de symétries géodésiques.

1. Introduction

Let $\Omega = G/K$ be an irreducible bounded symmetric domain in \mathbf{C}^d in its Harish-Chandra realization (i.e. a Cartan domain), with rank r and characteristic multiplicities a and b . Here G is the identity connected component

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of the biholomorphic automorphism group $\text{Aut}(\Omega)$ of Ω , and $K \subset G$ the subgroup stabilizing the origin $0 \in \Omega$. Under the action of G , the topological boundary $\partial\Omega$ has a decomposition

$$\partial\Omega = \partial_1\Omega \cup \dots \cup \partial_r\Omega$$

into G -orbits; each $\partial_l\Omega$, $l = 1, \dots, r$, is a disjoint union of *boundary faces*, which are also Cartan domains in their own right, except that they are of lower dimension and have their center not at the origin but at some point $v \in \partial_l\Omega$. The group G acts on $\partial\Omega$ by mapping the face $\Omega(v)$ centered at $v \in \partial_l\Omega$ into $\Omega(\tilde{v})$ with some $\tilde{v} \in \partial_l\Omega$. Also, the Cartan domain $\Omega(v)$, $v \in \partial_l\Omega$, has the same multiplicities a, b as Ω , and rank $r - l$; in particular, if $l = r$ then $\Omega(v)$ reduces to a point, and $\partial_r\Omega$ is exactly the Shilov boundary of Ω .

For any K -invariant finite measure μ on Ω which is absolutely continuous with respect to the Lebesgue measure, consider the convolution operator

$$(1.1) \quad B_\mu : f \mapsto f * \mu$$

acting on functions on Ω . That is,

$$(1.2) \quad B_\mu f(x) := \int_\Omega f \circ \phi \, d\mu \quad \text{where } \phi \in G, \phi(0) = x.$$

Owing to the K -invariance of μ , the right-hand side does not depend on the choice of ϕ satisfying $\phi(0) = x$, so the definition is unambiguous. One can take for ϕ e.g. the geodesic symmetry $\phi_x \in G$ interchanging x and the origin, or the *transvection* g_x defined by $g_x(z) := \phi_x(-z)$. Note that for

$$(1.3) \quad d\mu(z) = c_\nu h(z, z)^{\nu-p} \, dz,$$

where $h(x, y)$ is the Jordan triple determinant, $p = (r - 1)a + b + 2$ is the genus of Ω , dz stands for the Lebesgue measure, and c_ν is the normalization constant making $d\mu$ a probability measure, the operator B_μ coincides with the celebrated *Berezin transform* corresponding to the Wallach parameter $\nu > p - 1$ ([5], [12], [14], [3], [16], [6]).

In [10] it has been shown that if $x \rightarrow a \in \partial\Omega$, then $g_x \rightarrow g_a$, locally uniformly on Ω , where the limit g_a is a holomorphic retraction of Ω onto the boundary face containing a . Further, if $a \in \Omega(v)$ and $\alpha = a - v$, then $g_a = g_v g_\alpha = g_\alpha g_v$, where in the last occurrence g_α is understood in the Cartan domain $\Omega(v)$ rather than in Ω .

In [1], two of the present authors showed that the existence of the retraction g_a has important consequences for the boundary behaviour of the convolution operators B_μ . Namely, whenever f is a continuous function on Ω which extends continuously to $\Omega \cup \Omega(v)$, then the convolution $B_\mu f = f * \mu$

is also continuous on $\Omega \cup \Omega(v)$; further, the restriction of $B_\mu f$ to $\Omega(v)$ depends only on the restriction of f to $\Omega(v)$, and the operator

$$(1.4) \quad f|_{\Omega(v)} \mapsto (B_\mu f)|_{\Omega(v)}$$

is again an operator of the form (1.1), except that the convolution is taken in $\Omega(v)$ rather than in Ω and in the place of μ there is some K_v -invariant finite measure μ_v on $\Omega(v)$ (K_v being the K -group for $\Omega(v) \cong G_v/K_v$).

The actual determination of the measures μ_v from μ and v remained an open problem in [1]; in particular, it was conjectured there that for the case (1.3) of the Berezin transforms, the operators (1.1) are also of that type, though possibly with different ν .

The aim of this note is to prove the last conjecture in full: we exhibit an explicit formula relating μ and μ_v , which implies in particular that if μ is of the form (1.3) and $v \in \partial_t \Omega$, then μ_v is also of the form (1.3) — taken in the Cartan domain $\Omega(v)$ instead of Ω — except that ν gets replaced by $\nu - \frac{la}{2}$. The proof goes by transferring everything, via the Cayley transform, from the bounded domain Ω into its unbounded realization as Siegel domain of type II, where additional computational machinery is available. This is done in Section 2.

Our second result is that the unbounded realization also yields a very simple formula for the holomorphic retraction g_v : namely, upon conjugation with the Cayley transform, g_v becomes simply the orthogonal projection onto (the image of) the corresponding boundary face $\Omega(v)$. This is proved in Section 3, where also the special case of matrix balls — i.e. of Cartan domains of type I in the notation of Hua’s book [9] — is worked out as an example.

2. The measures μ_v

As in the Introduction let $\Omega = G/K$ be a Cartan domain in \mathbf{C}^d of type (r, a, b) , given in its Harish-Chandra realization; and let ϕ_x and g_x , $x \in \Omega$, be the geodesic symmetries interchanging x and the origin and the transvections $g_x(z) = \phi_x(-z)$, respectively.

We will use the language of Jordan theory, see e.g. [11], [7] or [2] for the details and notation. In particular, we let $Z (\cong \mathbf{C}^d)$ stand for the JB*-triple whose unit ball is Ω , $\{xyz\}$ for the triple product of Z , $D(x, y)$ for the multiplication operators $D(x, y)z = \{xyz\}$, and $Q(x)$ for the (antilinear) quadratic operator $Q(x)z = \{xzx\}$. An element $v \in Z$ is a *tripotent* if $\{vvv\} = v$. Two tripotents u, v are said to be *orthogonal* if $D(u, v) = 0$

(this is equivalent to $D(v, u) = 0$). Associated to a tripotent v is the *Peirce decomposition*

$$(2.1) \quad Z = Z_1(v) \oplus Z_{1/2}(v) \oplus Z_0(v),$$

with $Z_\nu(v) = \text{Ker}(D(v, v) - \nu)$ for $\nu = 0, \frac{1}{2}, 1$. Each $Z_\nu(v)$ is a subtriple of Z , and $Z_1(v)$ is a JB*-algebra under the product $x \circ y = \{xvy\}$, with unit v and involution $z^* = \{vzv\}$. A tripotent v is called *minimal* if $\dim Z_1(v) = 1$ and is called *maximal* if $Z_0(v) = 0$.

To a system e_1, \dots, e_m of pairwise orthogonal tripotents, there is similarly associated the *joint Peirce decomposition*

$$(2.2) \quad Z = \bigoplus_{0 \leq i \leq j \leq m} Z_{ij}$$

of Z into subspaces

$$(2.3) \quad Z_{ij} = \left\{ z \in \mathbf{C}^d : D(e_k, e_k)z = \frac{\delta_{ik} + \delta_{jk}}{2} z_0, \forall k = 1, \dots, m \right\},$$

of which (2.1) is a special case (for $m = 1$).

Any maximal system of pairwise orthogonal minimal tripotents e_1, \dots, e_r is called a *frame*; its cardinality r is the same for all frames in Z and equal to the *rank* of Ω . The characteristic multiplicities

$$a := \begin{cases} 2, & r = 1 \\ \dim Z_{12}, & \text{otherwise} \end{cases}$$

and

$$b := \dim Z_{01}$$

then depend neither on the ordering nor on the choice of the frame e_1, \dots, e_r .

Any $z \in Z$ can be written in the form (called the “polar-spectral decomposition” of z)

$$z = k(t_1 e_1 + \dots + t_r e_r),$$

where $k \in K$ and $t_1 \geq t_2 \geq \dots \geq 0$. The numbers t_1, \dots, t_r depend only on z (but not on the Jordan frame e_1, \dots, e_r used) and generalize the singular values of rectangular matrices. Further, z belongs to Ω , $\partial\Omega$ or $\partial_l\Omega$ ($l = 1, \dots, r$), respectively, if and only if $t_1 < 1$, $t_1 = 1$, or $1 = t_1 = \dots = t_l > t_{l+1} \geq \dots \geq t_r$; and z is a tripotent in $\partial_l\Omega$ (or, a tripotent of *rank* l) if and only if $t_1 = \dots = t_l = 1$, $t_{l+1} = \dots = t_r = 0$. For any such tripotent, the intersection

$$\Omega_0(v) := \Omega \cap Z_0(v)$$

is a Cartan domain of type $(r - l, a, b)$, and its translate

$$\Omega(v) := v + \Omega_0(v)$$

is precisely the boundary face centered at v . (The closure $\overline{\Omega(v)}$ is a face in the sense of convex geometry, *i.e.* intersection of $\overline{\Omega}$ with a supporting real hyperplane in Z . Further, $\Omega(v)$ is also a “holomorphic arc component” in the sense of being a maximal set whose points can be connected to one another by a chain of holomorphic images of the disc. See [11], Chapter 6.) All boundary faces arise in this way. The element

$$e := e_1 + e_2 + \dots + e_r$$

is a maximal tripotent.

Recall that in any Jordan algebra J with unit e and product $x \circ y$ an element x is called *invertible* if it has a (necessarily unique) inverse $y =: x^{-1}$ satisfying $x \circ y = e$ and $x^2 \circ y = x$. In the special case that the Jordan algebra arises as $J = Z_1(e)$ for a tripotent of the JB*-triple Z , the invertibility of $z \in J$ is equivalent to the invertibility of the operator $Q(z)$ on J and then $z^{-1} = Q(z)^{-1}Q(e)z$. In particular, taking the inverse is a rational map on J that can be written (see *e.g.* [15, Chapter 4]) in exact (*i.e.* reduced) form as $z^{-1} = p(z)/N(z)$, where $p : J \rightarrow J$ is a polynomial which generalizes the matrix adjoint and $N : J \rightarrow \mathbf{C}$ is a polynomial called the *determinant polynomial*, or *Koecher norm*, of the Jordan algebra. In particular, fixing a Jordan frame e_1, \dots, e_r of Z the above applies to the Jordan algebras $Z_1(e_1 + \dots + e_j)$, $1 \leq j \leq r$; we denote the corresponding determinant polynomials by N_j and extend them to all of Z by defining $N_j(z) := N_j(P_{1j}(z))$, where P_{1j} is the canonical projection of Z onto $Z_1(e_1 + \dots + e_j)$ given by the Peirce decomposition (2.1). For an r -tuple $\mathbf{m} = (m_1, \dots, m_r)$ of integers satisfying $m_1 \geq m_2 \geq \dots \geq m_r \geq 0$, the *conical polynomial* $N^{\mathbf{m}}$ associated with \mathbf{m} is

$$N^{\mathbf{m}} := N_1^{m_1 - m_2} N_2^{m_2 - m_3} \dots N_r^{m_r}.$$

In particular,

$$N^{\mathbf{m}} \left(\sum_{j=1}^r t_j e_j \right) = \prod_{j=1}^r t_j^{m_j}.$$

For all $z \in Z$ and all $\lambda = (\lambda_1, \dots, \lambda_r) \in \mathbf{C}^r$ such that for some integer l

$$N_j(z) > 0, \quad \forall j \leq l \quad \text{and} \quad \lambda_k = 0, \quad \forall k > l$$

we can even define

$$(2.4) \quad N^\lambda(z) := \prod_{j=1}^l N_j^{\lambda_j - \lambda_{j+1}}(z),$$

where for convenience we have put $\lambda_{r+1} := 0$. This applies, in particular, for $l = r$ to all z in the convex cone

$$\Lambda := \{ \exp x : x = x^* \in Z_1(e) \}$$

of positive elements in $Z_1(e)$ since $N_j > 0$ on Λ for all j .

The Cayley transform $\gamma = \gamma^{[\Omega, e]}$ associated with the domain Ω and a maximal tripotent $e \in Z$ is defined by⁽¹⁾

$$(2.5) \quad \begin{aligned} \gamma(z) &= (e - z_1) \circ (e + z_1)^{-1} - 2\{(e + z_1)^{-1}, e, z_{1/2}\} \\ &= \{e - z_1 - 2z_{1/2}, e, (e + z_1)^{-1}\}, \end{aligned}$$

for $z = z_1 + z_{1/2} \in Z_1(e) \oplus Z_{1/2}(e) = Z$. Let $F : Z_{1/2}(e) \times Z_{1/2}(e) \rightarrow Z_1(e)$ be the $Z_1(e)$ -valued Hermitian form

$$F(x, y) := \{x, y, e\},$$

and define

$$\tau(z) := \frac{z_1 + z_1^*}{2} - F(z_2, z_2).$$

Then the Cayley transform (2.5) maps Ω biholomorphically onto the Siegel domain

$$(2.6) \quad \mathcal{S} := \{z \in Z : \tau(z) \in \Lambda\}.$$

Its inverse is given by

$$\begin{aligned} \gamma^{-1}(w) &= (e - w_1) \circ (e + w_1)^{-1} - 4\{(e + w_1)^{-1}, e, w_2\} \\ &= \{e - w_1 - 4w_2, e, (e + w_1)^{-1}\}. \end{aligned}$$

The following lemma is immediate from the definition (2.5) of γ . Note that γ maps e into 0 and 0 into e .

LEMMA 2.1. — *Let $e = e_1 + \dots + e_r$ be a maximal tripotent and $v = e_1 + \dots + e_l$ a tripotent of rank l . Then for any $x \in Z_0(v)$,*

$$\begin{aligned} \gamma(v + x) &= 0 + \gamma^{[\Omega_0(v), e-v]}(x), \\ \gamma(0 + x) &= v + \gamma^{[\Omega_0(v), e-v]}(x), \\ \gamma^{-1}(v + x) &= 0 + (\gamma^{[\Omega_0(v), e-v]})^{-1}(x), \\ \gamma^{-1}(0 + x) &= v + (\gamma^{[\Omega_0(v), e-v]})^{-1}(x), \end{aligned}$$

where $\gamma^{[\Omega_0(v), e-v]}$ stands for the Cayley transform associated with the Cartan domain $\Omega_0(v)$ and its maximal tripotent $e - v$.

⁽¹⁾The Cayley transform defined here differs slightly from the one used by many other authors (that will also occur in Section 3).

In particular, γ maps the boundary face $\Omega(v)$ of Ω biholomorphically onto the boundary face $\mathcal{S}_0(v)$ of \mathcal{S} , where

$$\mathcal{S}_0(v) = \text{the interior of } \overline{\mathcal{S}} \cap Z_0(v)$$

(where the interior is understood in $Z_0(v)$, and the bar over \mathcal{S} denotes the closure).

It has been proved in [14] (see also [4]) that for “any” linear operator T on $C^\infty(\Omega)$ which commutes with G , i.e.

$$T(f \circ \phi) = (Tf) \circ \phi, \quad \forall \phi \in G,$$

the functions

$$e_\lambda(z) := N^\lambda(\tau(\gamma(z))), \quad z \in \Omega, \lambda \in \mathbf{C}^r,$$

are eigenfunctions of T :

$$(2.7) \quad Te_\lambda = \tilde{T}(\lambda)e_\lambda,$$

for any λ for which e_λ belongs to the domain of T . This applies, in particular, to all G -invariant differential operators T on Ω , as well as to all convolution operators $T = B_\mu$ of the form (1.1) with K -invariant finite measures μ . For the former, Te_λ is defined for any $\lambda \in \mathbf{C}^r$ and the map $T \mapsto \tilde{T}(\lambda + \rho)$, with ρ defined by

$$\rho_j = \frac{j-1}{2}a + \frac{b+1}{2}, \quad j = 1, \dots, r,$$

is known as the *Harish-Chandra isomorphism*; its image consists precisely of all polynomials on \mathbf{C}^r invariant under the Weyl group \mathfrak{W} generated by all permutations of the coordinates $\lambda_1, \dots, \lambda_r$ and the sign change $\lambda_1 \mapsto -\lambda_1$. For $T = B_\mu$, we will write just $\tilde{\mu}(\lambda)$ instead of $\tilde{T}(\lambda)$; note that in view of the K -invariance of μ , we then have $Te_\lambda = T\phi_\lambda$ where ϕ_λ are the *spherical functions*

$$\phi_\lambda(z) := \int_K e_\lambda(kz) dk,$$

dk being the normalized Haar measure on K . It is known that, unlike e_λ which is always unbounded, ϕ_λ is a bounded function on Ω whenever λ belongs to the set

$$W := \text{the closed convex hull of } \{\rho + \pi\rho : \pi \in \mathfrak{W}\}.$$

Since μ is assumed to be finite, $\tilde{\mu}$ is thus defined at least for $\lambda \in W$ (in particular — on some open set containing ρ) and is analytic there. Finally, for μ of the form (1.3) (so that B_μ is a Berezin transform), the eigenvalues

were computed explicitly by Unterberger and Upmeyer [14] (see also [4]): the result is

$$(2.8) \quad \tilde{\mu}(\lambda) = \prod_{j=1}^r \frac{\Gamma(\nu + \lambda_j - \frac{d}{r} - \frac{j-1}{2}a)\Gamma(\nu + 2\rho_j - \lambda_j - \frac{d}{r} - \frac{j-1}{2}a)}{\Gamma(\nu - \frac{d}{r} - \frac{j-1}{2}a)\Gamma(\nu + 2\rho_j - \frac{d}{r} - \frac{j-1}{2}a)}.$$

THEOREM 2.2. — *Let μ be a K -invariant measure on Ω , absolutely continuous with respect to the Lebesgue measure, v a tripotent of rank l , $\Omega(v)$ the boundary face with center v and μ_v the associated measure on the Cartan domain $\Omega_0(v)$. Then*

$$\tilde{\mu}_v(\lambda_1, \dots, \lambda_{r-l}) = \tilde{\mu}(\lambda_1, \dots, \lambda_{r-l}, 0, \dots, 0)$$

for all $(\lambda_1, \dots, \lambda_{r-l}, 0, \dots, 0) \in W$.

Note that in view of the injectivity of the Harish-Chandra transform $\mu \mapsto \tilde{\mu}$, the formula in Theorem 2.2 determines the measure μ_v uniquely.

Proof. — Choosing a suitable Jordan frame, we may assume that $v = e_{r-l+1} + \dots + e_r$. Let γ and \mathcal{S} be the Cayley transform and the Siegel domain, respectively, associated to the maximal tripotent $e = e_1 + \dots + e_r$. We will use the subscript $_{[v]}$ or the superscript $^{[v]}$ to denote objects corresponding to the Cartan domain $\Omega_0(v)$ and the boundary face $\Omega(v) = v + \Omega_0(v)$ instead of Ω ; in particular, the ambient complex space is $Z_{[v]} = Z_0(v)$, the element

$$e_{[v]} := e - v = e_1 + \dots + e_{r-l}$$

is a maximal tripotent of $\Omega_0(v)$, and the Cayley transform $\gamma_{[v]}$ associated to $\Omega_0(v)$ and $e_{[v]}$ coincides with the $\gamma^{[\Omega_0(v), e-v]}$ from Lemma 2.1, which also relates it to the Cayley transform (2.5) associated with Ω and e . The same lemma also shows that the corresponding Siegel domain $\mathcal{S}_{[v]} = \gamma_{[v]}(\Omega_0(v))$ coincides with the boundary face $\gamma(\Omega(v)) = \mathcal{S}_0(v)$ of \mathcal{S} . Further, $Z_{1/2}^{[v]}(e_{[v]}) = Z_0(v) \cap Z_{1/2}(e)$ and $F(x, y) = \{x, y, e\} = \{x, y, e_{[v]}\} = F_{[v]}(x, y)$ for $x, y \in Z_0(v) \cap Z_{1/2}(e)$; it follows that $\tau_{[v]}$ is just the restriction of τ to $Z_0(v)$, and its image is contained in $Z_0(v) \cap Z_1(e)$. Finally, the Jordan algebras $Z_1(e_1 + \dots + e_j)$, $1 \leq j \leq r - l$, are the same for $\Omega_0(v)$ as for Ω , since they are contained in $Z_0(v) \cap Z_1(e)$. It follows that

$$(2.9) \quad N_j^{[v]}(\tau_{[v]}(w)) = N_j(\tau(w)) > 0, \quad \forall j = 1, \dots, r - l, \forall w \in \mathcal{S}_0(v),$$

and

$$N_{[v]}^{(\lambda_1, \dots, \lambda_{r-l})}(\tau_{[v]}(w)) = N^{(\lambda_1, \dots, \lambda_{r-l}, 0, \dots, 0)}(\tau(w)), \quad \forall w \in \mathcal{S}_0(v)$$

where the right-hand side makes sense in view of (2.4) and (2.9).

Combining this with the facts about the Cayley transforms mentioned a few lines above and with Lemma 2.1, we get

$$(2.10) \quad \begin{aligned} e_{\lambda}^{[v]}(x) &= N_{[v]}^{\lambda}(\tau_{[v]}(\gamma_{[v]}(x))) \\ &= N^{(\lambda,0)}(\tau(\gamma(v+x))) = e_{(\lambda,0)}(v+x), \quad \forall x \in \Omega_0(v), \end{aligned}$$

where, for brevity, we write $(\lambda_1, \dots, \lambda_{r-l}) = \lambda$ and $(\lambda_1, \dots, \lambda_{r-l}, 0, \dots, 0) = (\lambda, 0)$, and $e_{(\lambda,0)}$ is extended continuously to $\Omega(v)$ via (2.9). Consequently, for any $x \in \Omega_0(v)$,

$$\begin{aligned} \tilde{\mu}_v(\lambda)e_{\lambda}^{[v]}(x) &= (B_{\mu_v}^{[v]}e_{\lambda}^{[v]})(x) \quad \text{by (2.7)} \\ &= (B_{\mu}e_{(\lambda,0)})(v+x) \quad \text{by the definition of } \mu_v \\ &= \tilde{\mu}(\lambda, 0)e_{(\lambda,0)}(v+x) \quad \text{by (2.7) again} \\ &= \tilde{\mu}(\lambda, 0)e_{\lambda}^{[v]}(x). \end{aligned}$$

Since $e_{\lambda}^{[v]}$ does not vanish identically on $\Omega_0(v)$ (for instance, $e_{\lambda}^{[v]}(0) = 1$), we must have $\tilde{\mu}_v(\lambda) = \tilde{\mu}(\lambda, 0)$, which proves the theorem. □

COROLLARY 2.3. — *If μ is of the form (1.3) for some $\nu > p - 1$, then μ_v is of the same form (with respect to $\Omega_0(v)$) only with ν replaced by $\nu - \frac{la}{2}$, where $l = \text{rank } v$.*

Proof. — For μ as in (1.3) we have by (2.8)

$$(2.11) \quad \begin{aligned} \tilde{\mu}(\lambda_1, \dots, \lambda_{r-l}, 0, \dots, 0) \\ &= \prod_{j=1}^r \frac{\Gamma(\nu + \lambda_j - \frac{d}{r} - \frac{j-1}{2}a)\Gamma(\nu + 2\rho_j - \lambda_j - \frac{d}{r} - \frac{j-1}{2}a)}{\Gamma(\nu - \frac{d}{r} - \frac{j-1}{2}a)\Gamma(\nu + 2\rho_j - \frac{d}{r} - \frac{j-1}{2}a)} \\ &= \prod_{j=1}^{r-l} \frac{\Gamma(\nu + \lambda_j - \frac{d}{r} - \frac{j-1}{2}a)\Gamma(\nu + 2\rho_j - \lambda_j - \frac{d}{r} - \frac{j-1}{2}a)}{\Gamma(\nu - \frac{d}{r} - \frac{j-1}{2}a)\Gamma(\nu + 2\rho_j - \frac{d}{r} - \frac{j-1}{2}a)}. \end{aligned}$$

On the other hand, for $\Omega_0(v) =: \Omega^{[v]}$ in the place of Ω we have $r^{[v]} = r - l$, $a^{[v]} = a$, $b^{[v]} = b$, so $\rho_j^{[v]} = \rho_j$ while

$$\frac{d^{[v]}}{r^{[v]}} = \frac{r^{[v]} - 1}{2}a^{[v]} + b^{[v]} + 1 = \frac{r - l - 1}{2}a + b + 1 = \frac{d}{r} - \frac{la}{2}.$$

Thus for a measure η of the form (1.3) but on $\Omega_0(v)$ and with σ in the place of ν we have

$$\begin{aligned}
 (2.12) \quad & \tilde{\eta}(\lambda_1, \dots, \lambda_{r-l}) \\
 &= \prod_{j=1}^{r-l} \frac{\Gamma(\sigma + \lambda_j - \frac{d^{[v]}}{r^{[v]}} - \frac{j-1}{2}a^{[v]})\Gamma(\sigma + 2\rho_j^{[v]} - \lambda_j - \frac{d^{[v]}}{r^{[v]}} - \frac{j-1}{2}a^{[v]})}{\Gamma(\sigma - \frac{d^{[v]}}{r^{[v]}} - \frac{j-1}{2}a^{[v]})\Gamma(\sigma + 2\rho_j^{[v]} - \frac{d^{[v]}}{r^{[v]}} - \frac{j-1}{2}a^{[v]})} \\
 &= \prod_{j=1}^{r-l} \frac{\Gamma(\sigma + \lambda_j - \frac{d}{r} + \frac{la}{2} - \frac{j-1}{2}a)\Gamma(\sigma + 2\rho_j - \lambda_j - \frac{d}{r} + \frac{la}{2} - \frac{j-1}{2}a)}{\Gamma(\sigma - \frac{d}{r} + \frac{la}{2} - \frac{j-1}{2}a)\Gamma(\sigma + 2\rho_j - \frac{d}{r} + \frac{la}{2} - \frac{j-1}{2}a)}.
 \end{aligned}$$

Comparing (2.11) and (2.12), we see that $\tilde{\mu} = \tilde{\eta}$ if $\nu = \sigma + \frac{la}{2}$. Since the function $\tilde{\eta}$ determines η uniquely, this completes the proof. \square

Note that since $\rho^{[v]} = \rho$, we have $W_{[v]} = W \cap (C^{r-l} \times \{0\})$, so that $(\lambda, 0) \in W$ whenever $\lambda \in W_{[v]}$.

We also remark that the relation $\sigma = \nu - \frac{la}{2}$ can be rewritten as

$$\nu - \frac{p-1}{2} = \sigma - \frac{p^{[v]}-1}{2}.$$

In particular, $\nu > p-1$ implies that $\sigma > \frac{p^{[v]}+p}{2} - 1 = p^{[v]} - 1 + \frac{la}{2} > p^{[v]} - 1$ as well.

3. The retraction ρ_v on Siegel domains

Consider again a bounded symmetric domain $\Omega \subset Z$ in its Harish-Chandra realization that is not necessarily irreducible in the sequel. Throughout we denote by $\sigma \in K \subset G$ the symmetry at the origin defined by $\sigma(z) = -z$. For every $a \in \overline{\Omega}$ we will describe in more detail the limit transvection

$$g_a := \lim_{\Omega \ni x \rightarrow a} g_x$$

on Ω by using a suitable Siegel domain realization. The image of g_a is known to be a holomorphic arc component $\Omega(v)$ of $\overline{\Omega}$ for some tripotent $v \in Z$. Because of $g_a = gg_v g^{-1}$ for a suitable $g \in G$ it is enough to study the special case $a = v$ for a fixed tripotent $v \in Z$ in the following. There exists an open connected neighbourhood $U \subset Z$ of $\Omega \cup \Omega(v)$ such that g_v extends to a holomorphic map $g_v : U \rightarrow U$ with $g_v = g_v^2$ and range $\Omega(v)$ — thus justifying the name “limit retraction”.

Consider the Peirce decomposition (2.1) associated to v and let $P_\nu = P_\nu(v) \in \text{End}(Z)$ stand for the canonical projection with range $Z_\nu(v)$ for $\nu = 1, 1/2, 0$. Also let $e \in Z$ be a maximal tripotent such that $v, e - v$ are orthogonal tripotents in Z . With $\gamma = \gamma^{[\Omega, e]}$ we denote the Cayley transform

as defined in Section 2. Then γ maps Ω biholomorphically onto the Siegel domain \mathcal{S} , see (2.6), and $\Omega(v)$ onto $\mathcal{S}_0(v)$. The main result of this section now states that g_v gets a very simple form when transformed to the Siegel domain \mathcal{S} .

THEOREM 3.1. — $\gamma g_v \gamma^{-1}$ on the Siegel domain \mathcal{S} is nothing but the Peirce-0-projection $P_0(v)$ restricted to \mathcal{S} .

Instead of Theorem 3.1 we prove the more general Theorem 3.2 below and start with some preliminaries: Denote by M be the compact dual of the bounded symmetric domain Ω . Then, in particular, M is a homogeneous complex manifold containing Z as a dense open subset and such that every $g \in \text{Aut}(\Omega)$ extends to a biholomorphic automorphism of M . In this sense we consider $\text{Aut}(\Omega)$ as a subgroup of $\text{Aut}(M)$. The identity connected component L of $\text{Aut}(M)$ is a complex Lie group containing G as a real form, that is, $\mathfrak{l} = \mathfrak{g} \oplus i\mathfrak{g}$ for the Lie algebra \mathfrak{l} of L .

Now for the vector field $\xi := (v + \{zvz\}) \partial/\partial z \in i\mathfrak{g} \subset \mathfrak{l}$,

$$(3.1) \quad \kappa_v := \exp\left(\frac{\pi}{4}\xi\right) \in L$$

is the *partial Cayley transform* defined by v (see Loos [11], Section 10). It maps Ω biholomorphically onto the Siegel domain of third kind

$$\mathcal{D} := \{z_1 \oplus z_{1/2} \oplus z_0 \in Z = Z_1(v) \oplus Z_{1/2}(v) \oplus Z_0(v) : \Re(z_1 - F_{z_0}(z_{1/2}, z_{1/2})) \in \Lambda_v \text{ and } z_0 \in \Omega\}.$$

Here $\Lambda_v \subset Z_1(v)$ is again the positive cone $\Lambda_v := \{\exp x : x = x^* \in Z_1(v)\}$ with $\Re(x) := (x + x^*)/2$ for $x \in Z_1(v)$. Also for every $z_0 \in Z_0(v)$, the sesquilinear map $F_{z_0} : Z_{1/2}(v) \times Z_{1/2}(v) \rightarrow Z_1(v)$ is defined by $F_{z_0}(y, y') = \{y, (I + \Phi_{z_0})^{-1}y', v\}$ with $\Phi_{z_0} \in \text{End}(Z_{1/2}(v))$ given by $y \mapsto 2\{eyz_0\}$.

The partial Cayley transform satisfies $\kappa_v^{-1} = \kappa_{-v}$ and $\kappa_v^8 = \text{id}_M$ for its eighth power. With respect to the Peirce decomposition $Z = Z_1(v) \oplus Z_{1/2}(v) \oplus Z_0(v)$ it is given by the explicit formula

$$(3.2) \quad \kappa_v(z_1 \oplus z_{1/2} \oplus z_0) = \{(v - z_1)^{-1}, v, v + z_1 + 2\sqrt{2}z_{1/2}\} + \{z_{1/2}, (v - z_1^*)^{-1}, z_{1/2}\} + z_0$$

provided $(v - z_1)$ is invertible in the unital Jordan algebra $Z_1(v)$.

As projective algebraic manifold M is rational and every $\lambda \in \text{End}(V)$ can be extended to a meromorphic selfmap of M that we also denote by λ , see Remark 3.3. In particular, every Peirce projection on Z can be considered as a meromorphic selfmap of M . Since g_v on Ω is given by

$$(3.3) \quad g_v(z) = v + z_0 - \{z_{1/2}, (v + z_1^*)^{-1}, z_{1/2}\}$$

(see [10] or [1]), g_v also can be considered as a meromorphic map on M . In this sense we can state the next result (where in contrast to Theorem 3.1 the tripotent e is not assumed to be maximal).

THEOREM 3.2. — *Let $u, v \in Z$ be orthogonal tripotents and $e := u + v$. Then g_v is a meromorphic selfmap of M with $g_v\kappa_e = \kappa_e P_0$, where $P_0 = P_0(v)$.*

Proof. — In a first step we consider the special case $u = 0$, that is $e = v$. Then $\sigma\kappa_v\sigma = \kappa_{-v}$ and by (3.2) we have

$$P_0\kappa_v\sigma(z) = \{z_{1/2}, (v + z_1^*)^{-1}, z_{1/2}\} - z_0$$

on Ω . On the other hand, by (3.2) and (3.3) we have

$$\kappa_v\sigma g_v(z) = -z_0 + \{z_{1/2}, (v + z_1^*)^{-1}, z_{1/2}\}.$$

This implies $P_0\kappa_v\sigma = \kappa_v\sigma g_v = \kappa_v g_{-v}\sigma$ and thus, replacing v by $-v$, the claim follows for the special case $u = 0$.

In the second step we consider the case $u \neq 0$. Since κ_u commutes with $D(v, v)$ it also commutes with $P_0 = P_0(v)$. But then

$$g_v\kappa_e = g_v\kappa_v\kappa_u = \kappa_v P_0\kappa_u = \kappa_v\kappa_u P_0 = \kappa_e P_0.$$

□

It remains to note that Theorem 3.2 implies Theorem 3.1 in case e is a maximal tripotent. Indeed, $\delta := P_1(e) + \sqrt{2}P_{1/2}(e)$ commutes with $P_0 = P_0(v)$ and $\kappa_e\sigma = \delta\gamma$ gives $\gamma g_v\gamma^{-1} = \delta^{-1}\kappa_e\sigma g_v\sigma\kappa_{-e}\delta = \delta^{-1}\kappa_e g_{-v}\kappa_{-e}\delta = \delta^{-1}P_0\delta = P_0$. □

Remark 3.3. — A few words concerning the notion of a “meromorphic mapping”, see also [13] p. 830: Suppose that M, N are connected complex manifolds and that $\mathcal{P}N$ is the power set of N , that is, the set of all subsets of N . We consider N in the canonical way as subset of $\mathcal{P}N$ by identifying every $x \in N$ with $\{x\} \in \mathcal{P}N$. Now a mapping $h : M \rightarrow \mathcal{P}N$ is called a *meromorphic map* from M to N if

- (i) The graph $\{(x, y) \in M \times N : y \in h(x)\}$ is a complex analytic subset of $M \times N$, that is, locally given by the solution set of a finite system of holomorphic equations.
- (ii) There is an open dense subset $U \subset M$ with $h(U) \subset N \subset \mathcal{P}N$.
- (iii) Γ is the closure of $\Gamma \cap (U \times N)$ in $M \times N$.

The irreducible bounded symmetric domains come in the six types I – VI. For the largest class among these, those of type I, we give a direct proof of Theorem 3.1 that does not use Jordan theory.

Example 3.4. — Fix integers $s \geq r \geq 1$ and let $Z := \mathbf{C}^{r \times s}$ be the space of all complex $r \times s$ -matrices (r rows and s columns). Then, if we denote in the following by I the unit matrix of any size and by $z^* = \bar{z}^t$ the conjugate transpose of the matrix z ,

$$\Omega := \{z \in Z : (I - zz^*) \text{ positive definite}\}$$

is a bounded symmetric domain in Z of rank r and with characteristic multiplicities $a = 2, b = s - r$. The corresponding triple product on Z is given by

$$\{x, y, z\} = \frac{xy^*z + zy^*x}{2}.$$

Considering every $\mathbf{C}^n \cong \mathbf{C}^{1 \times n}$ in the standard way as complex Hilbert space we also may interpret every $z \in Z$ as linear operator $z : \mathbf{C}^r \rightarrow \mathbf{C}^s$ via $z(x) := xz$, and then Ω becomes the open operator unit ball in Z . Also, the tripotents in Z are precisely the partial isometries and the maximal tripotents are precisely the isometries. For fixed integer l with $1 \leq l \leq r$ we write every $z \in Z$ as block matrix

$$z = \left[\begin{array}{cc|c} A & B & E \\ \hline C & D & F \end{array} \right]$$

with $A \in \mathbf{C}^{l \times l}, E \in \mathbf{C}^{l \times b}$ and all the other blocks of fitting sizes. Then

$$v := \left[\begin{array}{cc|c} I & 0 & 0 \\ \hline 0 & 0 & 0 \end{array} \right] \quad \text{and} \quad u := \left[\begin{array}{cc|c} 0 & 0 & 0 \\ \hline 0 & I & 0 \end{array} \right]$$

form a pair of orthogonal tripotents in Z such that $e := u + v$ is maximal (and, on the other hand, every other pair of orthogonal tripotents u', v' with $u' + v'$ maximal can be brought to this form with l suitable). The Peirce spaces $Z_1(e)$ and $Z_{1/2}(e)$ are characterized by the vanishing of the blocks E, F and A, B, C, D respectively. In particular, the JB*-algebra $Z_1(e)$ is isomorphic to $\mathbf{C}^{l \times l}$ with Jordan product $x \circ y = (xy + yx)/2$ and involution $x \mapsto x^* = \bar{x}^t$. Also the Jordan inverses in $Z_1(e)$ are the usual matrix inverses in $\mathbf{C}^{l \times l}$. It follows that the Cayley transform (2.5) associated with e is given by

$$(3.4) \quad \gamma(z) = \left[\begin{array}{cc|c} I + A & B \\ \hline C & I + D \end{array} \right]^{-1} \left[\begin{array}{cc|c} I - A & -B & -E \\ \hline -C & I - D & -F \end{array} \right] \text{ for all } z \in \Omega.$$

The Peirce components $z_1, z_{1/2}, z_0$ (with respect to v) of z in (3.3) are

$$z_1 = \left[\begin{array}{cc|c} A & 0 & 0 \\ \hline 0 & 0 & 0 \end{array} \right], \quad z_{1/2} = \left[\begin{array}{cc|c} 0 & B & E \\ \hline C & 0 & 0 \end{array} \right], \quad z_0 = \left[\begin{array}{cc|c} 0 & 0 & 0 \\ \hline 0 & D & F \end{array} \right].$$

Consequently,

$$\begin{aligned} g_v(z) &= \begin{bmatrix} I & 0 & | & 0 \\ 0 & D & | & F \end{bmatrix} - \begin{bmatrix} 0 & B & | & E \\ C & 0 & | & 0 \end{bmatrix} \begin{bmatrix} (I + A^*)^{-1} & 0 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}^* \begin{bmatrix} 0 & B & | & E \\ C & 0 & | & 0 \end{bmatrix} \\ &= \begin{bmatrix} I & 0 & | & 0 \\ 0 & D - d & | & F - f \end{bmatrix}, \end{aligned}$$

where

$$(3.5) \quad d := C(I + A)^{-1}B, \quad f := C(I + A)^{-1}E.$$

Thus by (3.4)

$$\begin{aligned} \gamma(g_v(z)) &= \begin{bmatrix} 2I & 0 \\ 0 & I + D - d \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & | & 0 \\ 0 & I - D + d & | & f - F \end{bmatrix} \\ (3.6) \quad &= \begin{bmatrix} 0 & 0 & | & 0 \\ 0 & (I + D - d)^{-1}(I - D + d) & | & (I + D - d)^{-1}(f - F) \end{bmatrix}. \end{aligned}$$

Setting

$$\begin{bmatrix} \eta & \theta \\ \alpha & \beta \end{bmatrix} := \begin{bmatrix} I + A & B \\ C & I + D \end{bmatrix}^{-1}$$

we have for the Peirce projection $P_0 = P_0(v)$

$$\begin{aligned} P_0(\gamma(z)) &= \begin{bmatrix} 0 & 0 \\ \alpha & \beta \end{bmatrix} \begin{bmatrix} 0 & -B & | & -E \\ 0 & I - D & | & -F \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & | & 0 \\ 0 & -\alpha B + \beta(I - D) & | & -\alpha E - \beta F \end{bmatrix}. \end{aligned}$$

Comparing with (3.6), we see that for the proof of $\gamma(g_v(z)) = P_0(\gamma(z))$ we only have to show that

$$(3.7) \quad \begin{aligned} -\alpha B + \beta(I - D) &= (I + D - d)^{-1}(I - D + d), \\ \alpha E + \beta F &= (I + D - d)^{-1}(F - f). \end{aligned}$$

As a shorthand we put $U := I + A$ and $V := I + D$. Then $\|A\| \leq \|z\| < 1$ implies that U^{-1} exists and thus, with $X := CU^{-1}$, $Y := U^{-1}B$,

$$\begin{bmatrix} U & B \\ C & V \end{bmatrix} = \begin{bmatrix} I & 0 \\ X & I \end{bmatrix} \begin{bmatrix} U & 0 \\ 0 & V - CU^{-1}B \end{bmatrix} \begin{bmatrix} I & Y \\ 0 & I \end{bmatrix}.$$

Consequently,

$$\begin{aligned} \begin{bmatrix} U & B \\ C & V \end{bmatrix}^{-1} &= \begin{bmatrix} I & -Y \\ 0 & I \end{bmatrix} \begin{bmatrix} U^{-1} & 0 \\ 0 & (V - CU^{-1}B)^{-1} \end{bmatrix} \begin{bmatrix} I & 0 \\ -X & I \end{bmatrix} \\ &= \begin{bmatrix} U^{-1} + Y(V - CU^{-1}B)^{-1}X & -Y(V - CU^{-1}B)^{-1} \\ -(V - CU^{-1}B)^{-1}X & (V - CU^{-1}B)^{-1} \end{bmatrix}. \end{aligned}$$

Hence

$$\beta = [I + D - C(I + A)^{-1}B]^{-1}, \quad \alpha = -\beta C(I + A)^{-1}.$$

By (3.5), $\beta = (I + D - d)^{-1}$. Consequently,

$$\begin{aligned} -\alpha B + \beta(I - D) &= \beta C(I + A)^{-1}B + \beta(I - D) \\ &= \beta(I - D + C(I + A)^{-1}B) \\ &= \beta(I - D + d) \\ &= (I + D - d)^{-1}(I - D + d), \end{aligned}$$

while

$$\begin{aligned} \alpha E + \beta F &= -\beta C(I + A)^{-1}E + \beta F \\ &= -\beta f + \beta F \\ &= (I + D - d)^{-1}(F - f). \end{aligned}$$

Thus $\gamma g_v = P_0 \gamma$, verifying Theorem 3.1 independently for the case of all Cartan domains Ω of type I. □

The compact dual M of the bounded symmetric domain $\Omega \subset Z = \mathbf{C}^{r \times s}$ is the Grassmannian $\mathcal{G}_{r,s}$ of all r -dimensional linear subspaces in \mathbf{C}^{r+s} . The embedding $Z \hookrightarrow M$ is given by identifying every $z \in Z$ with its graph

$$\{(x, xz) \in \mathbf{C}^{r+s} : x \in \mathbf{C}^r\} \in \mathcal{G}_{r,s}.$$

In particular, in case $r = s = 1$ the dual $M = \mathcal{G}_{1,1}$ is nothing but the Riemann sphere $\mathbf{C} \cup \{\infty\}$.

Remark 3.5. — The last example makes it possible to handle also some Cartan domains of types II – IV (in Hua’s notation [9]). Namely, every JB*-triple Z of type II – IV containing an invertible element can be reduced to the worked out type I case $W := \mathbf{C}^{n \times n}$ for suitable n . More concretely, for tripotents e, v as before such a Z can be realized as subtriple of W in such a way that e is the unit matrix and v has the block form $\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$ in W . Indeed, if Z is of type II, i.e. the space of symmetric matrices in $\mathbf{C}^{r \times r}$, then this is obvious. If Z is of type III, that is, the space of skew-symmetric matrices in $W = \mathbf{C}^{n \times n}$, n even, we may assume without loss of generality that e is a unitary matrix. Then the triple automorphism $z \mapsto ze^*$ of W maps Z to an isomorphic subtriple containing the identity matrix $I = ee^*$. Finally, for Z of type IV such an embedding can be obtained in terms of Pauli spin matrices, see Harris [8], page 19. □

For every tripotent $v \in Z$ let $\rho_v : \Omega \rightarrow Z$ be defined by

$$\rho_v(z) := g_v(z) - v.$$

Then ρ_v is a holomorphic retraction of Ω onto $\Omega_0(v) = \Omega \cap Z_0(v)$, see [1].

COROLLARY 3.6. — *Using again the notation $\gamma_{[v]}$ for the Cayley transform in $Z_0(v)$ associated with the Cartan domain $\Omega_0(v)$ and the maximal tripotent $e - v$, we have*

$$\rho_v = \gamma_{[v]}^{-1} \circ P_0(v) \circ \gamma.$$

Proof. — Immediate from Theorem 3.1 and Lemma 2.1. □

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Jonathan ARAZY
University of Haifa
Department of Mathematics
31905 Haifa (Israel)
jarazy@math.haifa.ac.il

Miroslav ENGLIŠ
Silesian University in Opava
Mathematics Institute
Na Rybníčku 1
74601 Opava (Czech Republic)
and
Mathematics Institute
Žitná 25
11567 Praha 1 (Czech Republic)
englis@math.cas.cz

Wilhelm KAUP
Universität Tübingen
Mathematisches Institut
Auf der Morgenstelle 10
72076 Tübingen (Germany)
kaup@uni-tuebingen.de