# ANNALES

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# L'INSTITUT FOURIER

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**Pierre and Sylvestre: some recollections**

# PIERRE AND SYLVESTRE: SOME RECOLLECTIONS

### by Gérard BESSON

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My first encounters with Pierre Bérard and Sylvestre Gallot were at the Geometry seminar in Paris VII. The seminar was called *le Séminaire Arthur Besse* and Marcel Berger was in charge of things. At that time I was looking for an advisor so I turned up, hoping to find one, for the latest talk. I sat there in the lecture room and, needless to say, did not understand a single word. However, I do remember clearly that a theorem on harmonic manifolds due to Alamigeon was cited. Just then, a young fellow at the back, who was taking notes enthusiastically, asked a question. Marcel Berger replied "Well, this is a question for you !". The young guy was Sylvestre who had not as yet defended his " thèse d'état " but was very close to doing so. Pierre, on the other hand, was not at the talk that day. He was visiting IMPA as part of his military service and so I first met him a couple of years later, again at the seminar Besse, when he returned to France. I had started to work on the spectrum of the Laplacian under the supervision of Marcel Berger, and after a lecture that I gave he kindly came up to me and we discussed my project. In those days Pierre had a beard, the one we see on the picture.

In 1978, Sylvestre defended his thesis. It was a tradition to give a second lecture after the defense, called " deuxième thèse ", which was more like a masters' thesis. Sylvestre's thesis was about inequalities for the eigenvalues of the Laplacian and lower bound on the Ricci curvature; some striking results were described which put Sylvestre's works in light. His " second " thesis was about Nash's embedding theorem and all I can remember is a slide full of Sobolev spaces and maps between them.

I really started to discuss mathematics with Pierre after his beautiful paper on the spectrum of certain domains in the Euclidean space related to

crystallographic groups. To my knowledge it was the first systematic computation of the spectrum of the Laplacian for both Dirichlet and Neumann boundary conditions for a wide class of Euclidean domains. One striking example is one half of an equilateral triangle for which the spectrum was not know at all before!

It was not so easy to get a position in France at that time; I remember one year with no assistant position and just around ten professor positions. Nevertheless, Sylvestre got hired in Chambéry in 1980 and Pierre got a job there too a year later. They started to work on a wonderful program on isoperimetric inequalities on Riemannian manifolds satisfying a lower bound on the Ricci curvature. Yves Colin de Verdière was in Grenoble and naturally there was a lot of activity on spectral theory of Riemannian manifold concentrated in the two towns which are only 60km apart. And so I moved to Grenoble in 1981. We had a joint seminar which would meet every week alternately in Chambéry or Grenoble. In 1983, we began publishing the proceedings of our seminar in which various works of Pierre and Sylvestre may be found. Their research culminated with the result on the estimate on the heat kernel which is an analytical translation of some isoperimetric inequalities *à la* Gromov. In 1984, I had just finished some work and was wondering what I should do next. They kindly invited me to join them in their project and what followed was an amazingly exciting time.

In 1986, Pierre moved to Grenoble. Sylvestre got jobs at the École Polytechnique and at the École Normale Supérieure de Lyon before finally moving to Grenoble in 1998. Roughly at the same time, in 1987, Sylvestre, Gilles Courtois and myself started working together on the entropy of symmetric spaces of negative curvature. Pierre began studying isospectrality of Riemannian manifolds, became one of the few renowned specialist in the world before turning to spectral problems in the theory of minimal surfaces theory. During this time, we all shared the same passion for Italy and developed fruitful exchanges with our colleagues there.

I omitted some fundamental aspects of Pierre and Sylvestre activities. Pierre has been involved in the development of mathematics at the highest levels, International Mathematical Union, French ministry of research, the "IST infrastructure cellule Math-Doc", University Joseph Fourier as Vice-President in charge of the Research, etc.. Sylvestre has devoted himself to a large number of graduate students, who have almost invariably turned out to be very successful.

This short historical account describes an exceptional and passionate journey through Mathematics and Riemannian Geometry in particular. The influence of both Pierre and Sylvestre's works is testified by the quality of the participant at the workshop. I wish to thank all of them as well as those who were eager to come but could not for their contribution to the success of the workshop. But above all it is the story of our friendship; no workshop nor proceedings could ever show how precious this friendship has been.

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